Trig 4.5

Approximate real zeros of a polynomial function
Use the upper and lower bound theorems

zero (of a function)

rational

real

Descarte's rule of signs

location principle

upper bound

lower bound

UB $f(x) \rightarrow$ no sign changes

LB $f(-x) \rightarrow$ opp. original $\rightarrow$ no sign changes

Use technology to help! But you still have to prove it!

graph...table

Approximate the real zeros of each function to the nearest tenth.

19. $f(x) = 3x^4 + x^2 - 1$

20. $f(x) = x^2 + 3x + 1$

x-coord.

Upper bound: There is only end behavior to the right of ...

(no max, min, zeros, etc.)

f(x) ...no sign changes in depressed polynomial

Upper Bound Theorem	Suppose c is a positive real number and $P(x)$ is divided by $x - c$. If the resulting quotient and remainder have no change in sign, then $P(x)$ has no real zero greater than c. Thus, c is an upper bound of the zeros of $P(x)$.

Zero coefficients are ignored when counting sign changes.

Lower bound: There is only end behavior to the left of... x-coord

(no max, min, zeros, etc.)

f(-c) ... no sign changes in depressed polynomial then c is a lower bound

(you try the opposite)

Goal: to zoom in as tightly as possible

A **lower bound** is an integer less than or equal to the least real zero. A lower bound of the zeros of $P(x)$ can be found by determining an upper bound for the zeros of $P(-x)$.

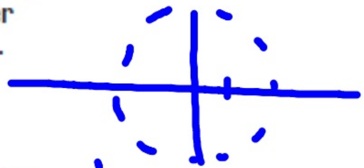
Lower Bound Theorem	If c is an upper bound of the zeros of $P(-x)$, then $-c$ is a lower bound of the zeros of $P(x)$.

UB = 1   LB = 0   UB = 2   LB = -1

Use the Upper Bound Theorem to find an integral upper bound and the Lower Bound Theorem to find an integral lower bound of the zeros of each function.

26. $f(x) = 3x^3 - 2x^2 + 5x - 1$ ± 1

27. $f(x) = x^2 - x - 1$ ± 1



$$\begin{array}{r|rrrr} -1 & 3 & -2 & 5 & -1 \\ & & 1 & -3 & \\ \hline & 3 & -5 & & \end{array}$$

$f(-x) = -3x^3 - 2x^2 + -5x - 1$

$$\begin{array}{r|rrrr} 0 & -3 & -2 & -5 & -1 \\ & \downarrow & 0 & 0 & 0 \\ \hline & -3 & -2 & -5 & -1 \end{array}$$

$f(-x) = x^2 + x - 1$

$$\begin{array}{r|rrr} 0 & 1 & 1 & -1 \\ & \downarrow & 0 & 0 \\ \hline & 1 & 1 & -1 \end{array}$$

WB 4.5 $_{\text{odds}}$ + 14