Trig 4.1

*Algebra 2 Ch. 5 & Ch. 7
Reminder: study guides on GCR

Determine roots of polynomial equations *
Apply the fundamental theorem of algebra (# of roots) *

factor ... x-factor
polynomial (in one variable)
degree
leading coefficient
polynomial function
zeros (real)
polynomial equation
Roots (can be real or imag.)
imaginary number $\sqrt{-1}$
real number
complex number $\underset{\text{real}}{2} + \underset{\text{imag.}}{3i}$
Fundamental Theorem of Algebra

$\underbrace{1000x^{18} + 500x^{10} + 250x^5}$ is a **polynomial in one variable.**

$x^2+6x+9=0$

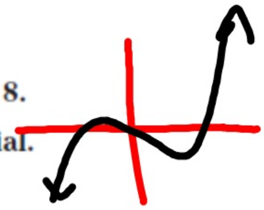
$\sqrt{-16} = \sqrt{16}\cdot\sqrt{-1} \rightarrow 4i$

$x^2+x-6=0$

2 Consider the polynomial function $f(x) = x^3 - 6x^2 + 10x - 8$.

a. State the degree and leading coefficient of the polynomial.

b. Determine whether 4 is a zero of $f(x)$.



$d = 3$

$LC = 1$

$f(4) = (4)^3 - 6(4)^2 + 10(4) - 8$

$= 64 - 96 + 40 - 8$

$f(4) = 0$

yes

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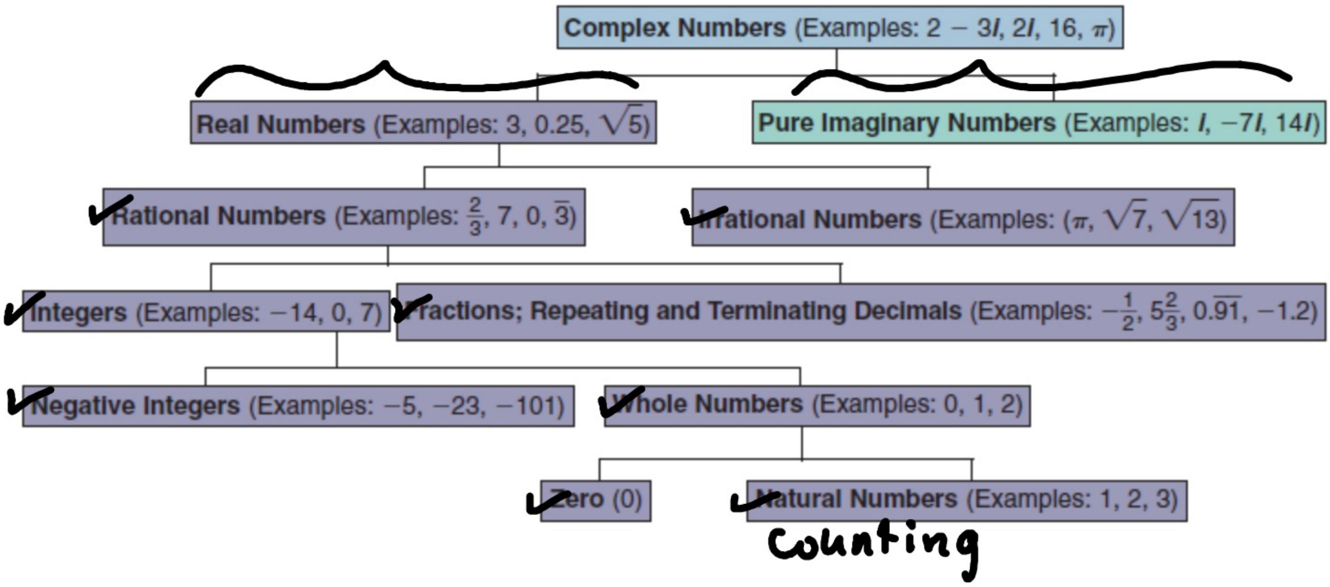

Complex Numbers (Examples: $2 - 3i$, $2i$, 16, π)
Real Numbers (Examples: 3, 0.25, $\sqrt{5}$)
Pure Imaginary Numbers (Examples: i, $-7i$, $14i$)
Rational Numbers (Examples: $\frac{2}{3}$, 7, 0, $\overline{3}$)
Irrational Numbers (Examples: (π, $\sqrt{7}$, $\sqrt{13}$)
Integers (Examples: -14, 0, 7)
Fractions; Repeating and Terminating Decimals (Examples: $-\frac{1}{2}$, $5\frac{2}{3}$, $0.\overline{91}$, -1.2)
Negative Integers (Examples: -5, -23, -101)
Whole Numbers (Examples: 0, 1, 2)
Zero (0)
Natural Numbers (Examples: 1, 2, 3)
counting

Fundamental Theorem of Algebra	Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers.

A corollary to the Fundamental Theorem of Algebra states that the degree of a polynomial indicates the number of possible roots of a polynomial equation.

Corollary to the Fundamental Theorem of Algebra

Every polynomial $P(x)$ of degree n ($n > 0$) can be written as the product of a constant k ($k \neq 0$) and n linear factors.

$$P(x) = k(x - r_1)(x - r_2)(x - r_3) \dots (x - r_n)$$

Thus, a polynomial equation of degree n has exactly n complex roots, namely $r_1, r_2, r_3, \dots, r_n$.

Degree = number of possible roots (...could be double root x=3 or x=3)
(might not be real)

Complex conjugate partners (alg 2)

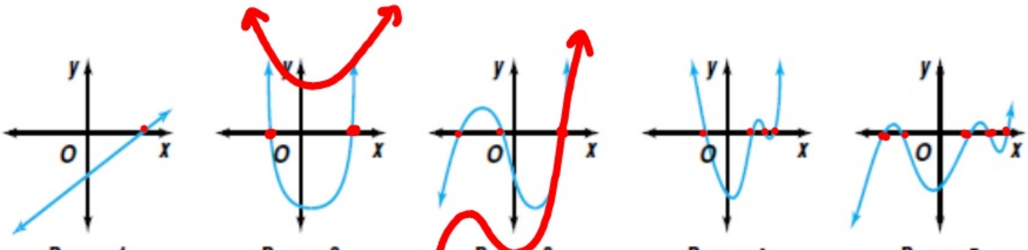
pairs

$3+2i \quad 3-2i$

$5-3i \quad 5+3i$

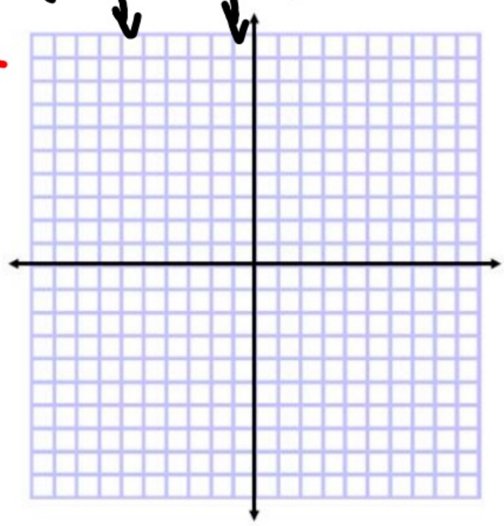
$7+\sqrt{3}i \quad 7-\sqrt{3}i$

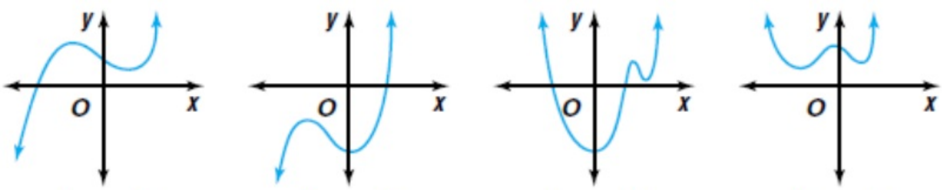
N roots: At most… (some could also be complex conjugate partner pairs)



Degree 1 | Degree 2 | Degree 3 | Degree 4 | Degree 5

$(x-1)(x-1)$





Degree 3, 1 x-intercept | Degree 3, 1 x-intercept | Degree 4, 2 x-intercepts | Degree 4, 0 x-intercepts

$a=1 \quad b=3 \quad c=6$

Solve the equation:

$x^2 + 3x + 6 = 0$

$x =$

$x =$

6

3

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$= \frac{-3 \pm \sqrt{(9-24)}}{2} = \frac{-3 \pm \sqrt{-15}}{2} =$$

-1

$$= \frac{3}{2} \pm \frac{i\sqrt{15}}{2}$$

$x^2 \sim\sim = 0$

Write the equation with roots x = 2 and x = -5
How many factors? Work backwards!

$x^2+3x-10=0$

$(x-2)(x+5)=0$

$x-2=0 \qquad x+5=0$

$x=2 \qquad x=-5$

$$\begin{array}{r} x+5 \\ x-2 \\ \hline -2x-10 \\ x^2+5x \\ \hline \end{array}$$

Complex conjugate partners...

$x = 2i \quad x = -2i$

$0 + 2i \quad 0 - 2i$

$x^2 \_\_\_\_\_ = 0$

$x^2 + 4 = 0$

$$\begin{array}{r} x - 2i \\ x + 2i \\ \hline +2ix - 4(ii) \\ x^2 - 2ix \\ \hline x^2 + 4 \end{array}$$

-1

$(x-2i)(x+2i) = 0$

$(x-2i) = 0 \quad (x+2i) = 0$

$x = 2i \quad x = -2i$

x^3 ______ $= 0$

3 a. Write a polynomial equation of least degree with roots 2, $4i$, and $-4i$.

b. Does the equation have an odd or even degree? How many times does the graph of the related function cross the x-axis? one

ⓐ $x^3 - 2x^2 + 16x - 32 = 0$

$(x-2)(x^2+16) = 0$

$(x-2)\overbrace{(x-4i)(x+4i)} = 0$

$x = 2$ $\quad x = 4i$ $\quad x = -4i$

$$\begin{array}{r} x-4i \\ x+4i \\ \hline -16i^2 \end{array}$$

$x^2 + 16$

$$\begin{array}{r} x^2+16 \\ x-2 \\ \hline -2x^2-32 \\ x^3 \quad +16x \\ \hline \end{array}$$

Did they tell you ALL the roots? They don't have to...

$y = x^2 - 1$

2

4 **State the number of complex roots of the equation $x^2 - 1 = 0$ = 0. Then find the roots and graph the related function.**

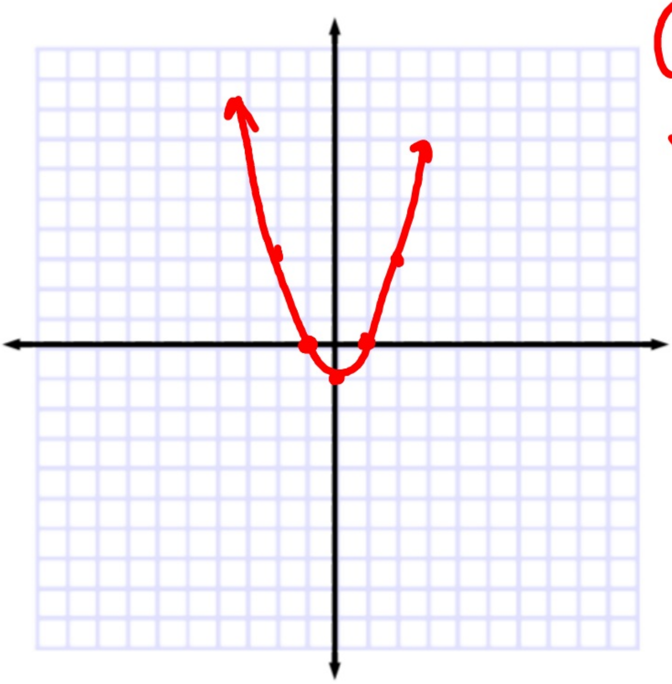
factor...

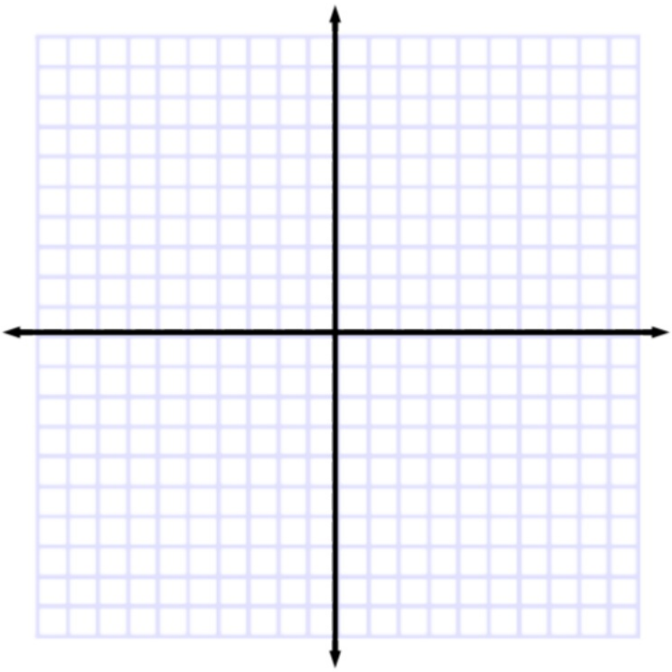
$x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x+1=0$ $\quad$ $x-1=0$

$x=-1$ $\quad$ $x=1$





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