Trig 7.4

2x

Use the double-angle identities for sine, cosine, tangent
Use the half-angle identities for sine, cosine, tangent

$\frac{1}{2}$

double mult x 2

half div x 2

$\sin(A+A) = \sin A \cos A + \cos A \sin A = 2 \sin A \cos A$

$\sin(2A)$

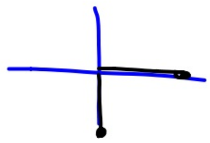
$\cos(A+A) = \cos A \cos A - \sin A \sin A = \cos^2 A - \sin^2 A$

$\tan(A+A) = \dfrac{\tan A + \tan A}{1 - \tan A \cdot \tan A} = \dfrac{2 \tan A}{1 - \tan^2 A}$

parking lot

(12)

$y = 20\sin(3t + \theta)$



$\star\ y = 20\sin(\overset{A}{3t} + \overset{B}{90^\circ})$ $\quad * \ y = 20\sin(\overset{A}{3t} + \overset{B}{270^\circ})$

$20(\sin 3t \cos 90 + \cos 3t \sin 90) + 20(\sin 3t \cos 270 + \cos 3t \sin 270)$

$20(\sin 3t) + 20(-\cos 3t)$

$20\sin 3t + -20\cos 3t$

$20(\sin 3t - \cos 3t)$

$(A - A)$

$\sin(A + A)$

$\cos(A + A)$

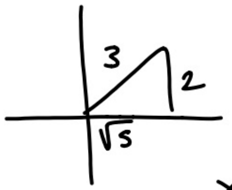
Double-Angle Identities	If θ represents the measure of an angle, then the following identities hold for all values of θ.
	$\sin 2\theta = 2 \sin\theta \cos\theta$
	$\cos 2\theta = \cos^2\theta - \sin^2\theta$
	$\cos 2\theta = 2\cos^2\theta - 1$
	$\cos 2\theta = 1 - 2\sin^2\theta$
	$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= 1 - \sin^2\theta - \sin^2\theta$$
$$= 1 - 2\sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - (1 - \cos^2\theta)$$
$$= \cos^2\theta - 1 + \cos^2\theta$$
$$= 2\cos^2\theta - 1$$

1 **If $\sin\theta = \frac{2}{3}$ and θ has its terminal side in the first quadrant, find the exact value of each function.**

a. $\sin 2\theta$



Which quadrant?
Reference triangle?
Find all sides
Which identity?

$x^2 + 2^2 = 3^2$

$x^2 + 4 = 9$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{\sqrt{5}}{3}$

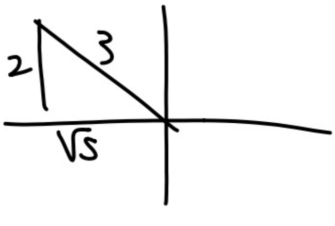
$= \frac{4\sqrt{5}}{9}$

b. $\cos 2\theta$ $= \cos^2\theta - \sin^2\theta$

$= \frac{\sqrt{5}}{3} \cdot \frac{\sqrt{5}}{3} - \frac{2}{3} \cdot \frac{2}{3} = \sqrt{25}$

$\frac{5}{9} - \frac{4}{9} = -\frac{1}{9}$

c. $\tan 2\theta$



$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} = \frac{\frac{2}{1} \cdot \frac{2}{\sqrt{5}}}{1 - \frac{2}{\sqrt{5}} \cdot \frac{2}{\sqrt{5}}}$

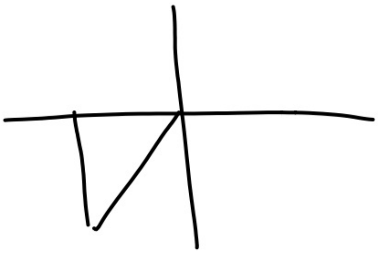
$= \frac{\frac{4}{\sqrt{5}}}{1 - \frac{4}{5}} = \frac{\frac{4}{\sqrt{5}}}{\frac{1}{5}} \cdot \frac{5}{1} = \frac{20}{\sqrt{5}}$

$= \frac{20}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{20\sqrt{5}}{5} = 4\sqrt{5}$

Use the given information to find sin 2θ, cos 2θ, and tan 2θ.

8. $\boxed{\sin\theta = \frac{2}{5}}, \boxed{0^\circ < \theta < 90^\circ}$

9. $\tan\theta = \frac{4}{3}, \boxed{\overset{180}{\pi} < \theta < \overset{270}{\frac{3\pi}{2}}}$



Which quadrant?
Reference triangle
Find all sides
Which identity?

$\cos 2\theta = 2\cos^2\theta - 1$

Solve for $\cos\theta$.

$+1 \qquad\qquad +1$

$$\frac{\cos 2\theta + 1}{2} = \frac{2\cos^2\theta}{2}$$

$$\sqrt{\frac{\cos 2\theta + 1}{2}} = \sqrt{\cos^2\theta}$$

$$\pm\sqrt{\frac{\cos 2\theta + 1}{2}} = \cos\theta$$

$\cos 2\theta = 1 - 2\sin^2\theta$ **Solve for sin θ.**

$-1 \quad -1$

$$\frac{\cos 2\theta - 1}{-2} = \frac{-2\sin^2\theta}{-2}$$

$$\sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\sin^2\theta}$$

$$\pm\sqrt{\frac{1-\cos 2\theta}{2}} = \sin\theta$$

x...2x

Half-Angle Identities	If α represents the measure of an angle, then the following identities hold for all values of α. $\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$ $\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$ $\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}, \cos\alpha \neq -1$

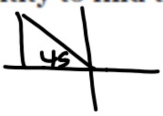
Unlike with the double-angles identities, you must determine the sign.

+ or - from original quadrant

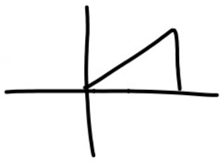
half/whole.....What is this angle half of? (should be a handy angle)
+ or - (from original question)

2 **Use a half-angle identity to find the exact value of each function.**

a. $\sin \frac{7\pi}{12}$



b. $\cos 67.5°$ $= +$



$\cos 67.5$

$\cos \frac{1}{2}(135) = +\sqrt{\frac{1+\cos 135}{2}}$

$= +\sqrt{\frac{\frac{2}{2} - \frac{\sqrt{2}}{2}}{2}} = +\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} \cdot \frac{1}{2}$

$= +\sqrt{\frac{2-\sqrt{2}}{4}} = \frac{+\sqrt{2-\sqrt{2}}}{2}$

15-310

half/whole

Use a half-angle identity to find the exact value of each function.

6. $\sin \frac{\pi}{8}$

7. $\tan 165^\circ$

4 **Verify that $\frac{\cos 2\theta}{1 + \sin 2\theta} = \frac{\cot \theta - 1}{\cot \theta + 1}$ is an identity.**

Both sides must match
Hey!