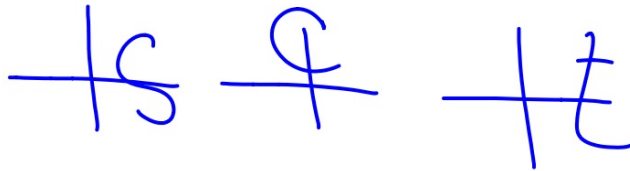Trig 7.5

Solve trig equations algebraically and graphically

~~Solve trig inequalities*~~

principal values

specific range

certain quadrants

all values (+360n or +2pi*n)

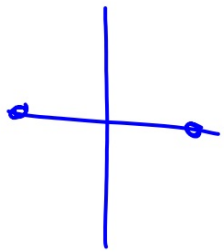verify (identities)

solve (equations)

graphing calculators p. 458

$$2 \sin 2\theta$$

29) $2 \sin\theta \cos\theta + \sqrt{3} \sin\theta = 0$ $0 \leq \theta < 360$

$\sin\theta \left(2\cos\theta + \sqrt{3}\right) = 0$

$\sin\theta = 0$ $2\cos\theta + \sqrt{3} = 0$ $150° \quad 210°$

$\boxed{0°, 180°}$ $2\cos\theta = -\sqrt{3}$

$\dfrac{2\cos\theta}{2} \quad \dfrac{-\sqrt{3}}{2}$

$\cos \qquad \theta = 30$

Strategies:
Use parking lot to get all same function
Factor etc. (alg 2)
Use zero product property
Which quadrant(s)?
Default radians (unless degrees is specified)

$$(\quad)(\quad) = 0$$

Principal values
(depends on which function)
sin
cos
tan

QF, CTS, factor...
zero product property

$30, 150$
$\frac{\pi}{6}$ $\frac{5\pi}{6}$

Solve each equation for $0 \le x < 2\pi$.

30. $(2 \sin x - 1)(2 \cos^2 x - 1) = 0$

31. $4 \sin^2 x + 1 = -4 \sin x$

$45, 315, 135, 225$
$\frac{\pi}{4}$ $\frac{7\pi}{4}$ $\frac{3\pi}{4}$ $\frac{5\pi}{4}$
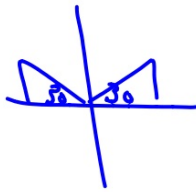
$\cos x = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$

$2\sin x - 1 = 0$ $2\cos^2 x - 1 = 0$

$2\sin x = 1$ $\frac{2\cos^2 x}{2} = \frac{1}{2}$

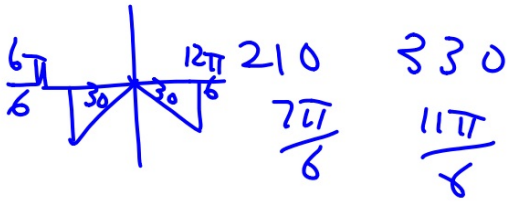$\frac{2\sin x}{2} = \frac{1}{2}$
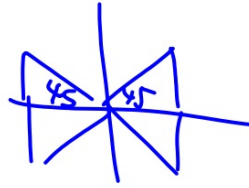
$\sin x = \frac{1}{2}$ $\sqrt{\cos^2 x} = \sqrt{\frac{1}{2}}$

$$4\sin^2 x + 1 = -4\sin x$$

$$4\sin^2 x + 4\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$\frac{6\pi}{6}$ $\frac{12\pi}{6}$ 210   330

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$

$$4a^2 + 4a + 1 = 0$$

$$(2a + 1)^2 = 0$$
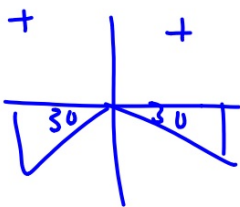
$$(2a+1)(2a+1) = 0$$

$$2a + 1 = 0$$

$$\frac{2a}{2} = -\frac{1}{2} \qquad a = -\frac{1}{2}$$

+ 2pi*n

Solve each equation for all real values of *x*.

37. $\sin x = -\dfrac{1}{2}$

+   +

30   30

210   330

$\dfrac{7\pi}{6} + 2\pi n$ $\dfrac{11\pi}{6} + 2\pi n$

$n \in \mathbb{Z}$

$2a^2 + a - 1 = 0$

$a = \dfrac{-1 \pm \sqrt{1 - 4 \cdot 2 \cdot -1}}{4} = \dfrac{-1 \pm \sqrt{9}}{4}$

$a = \dfrac{-1 \pm 3}{4}$

$\dfrac{2}{4} = \dfrac{1}{2}$

$\dfrac{-4}{4} = -1$

38. $\cos x \tan x - 2\cos^2 x = -1 + 2\cos x$

$\cos x \cdot \dfrac{\sin x}{\cos x} -$

$\sin x = -1 \, 2(1 - \sin^2 x)$

$= -1 + 2 - 2\sin^2 x$

$\sin x = 1 - 2\sin^2 x$

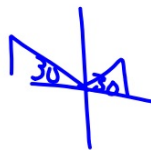$2\sin^2 x + \sin x - 1 = 0$

$\sin x = \dfrac{1}{2}$ $\sin x = -1$

30, 150   270   0,1

$\dfrac{\pi}{6} + 2\pi n$ $\dfrac{5\pi}{6} + 2\pi n$ $\dfrac{3\pi}{2} + 2\pi n$ 9-

$n \in \mathbb{Z}$

Graphing calc (make sure plots are OFF)

$y_1 = 2x + 3$

$y_2 = x^2$

Estimate the intersection(s)

CALC menu

$x^2 = 2x + 3$

2nd calc

intersect

1st   2nd

$\left[\, \bullet \,\right]$

guess

Check calculator mode (radians)...window
Turn plots OFF

GRAPHING CALCULATOR EXPLORATION

Some trigonometric equations and inequalities are difficult or impossible to solve with only algebraic methods. A graphing calculator is helpful in such cases.

TRY THESE   Graph each side of the equation as a separate function.

1. $\sin x = 2 \cos x$ for $0 \le x \le 2\pi$ ~~0~~ ~~6.28~~

2. $\tan 0.5x = \cos x$ for $-2\pi \le x \le 2\pi$

3. Use the **CALC** menu to find the intersection point(s) of the graphs in Exercises 1 and 2.

WHAT DO YOU THINK?

4. What do the values in Exercise 3 represent? How could you verify this conjecture?

5. Graph $y = 2 \cos x - \sin x$ for $0 \le x \le 2\pi$.

   a. How could you use the graph to solve the equation $\sin x = 2 \cos x$? How does this solution compare with those found in Exercise 3?

   b. What equation would you use to apply this method to $\tan 0.5x = \cos x$?

$y_1 = \sin x$ $y_2 = 2\cos x$

5. What happens when they are equal?

Graphing calc

Solve each equation graphically on the interval $0 \leq x < 2\pi$.

52. $\tan x = 0.5$ **53.** $\sin x - \frac{x}{2} = 0$ **54.** $\cos x = 3 \sin x$

WB 7.5 + p.460 53

skip < >