Trig 7.1

parking lot

Identify and use trig identities

* identity $2(x+3) = 2x+6$ always T

equation

counterexample $2x + 3 = 7$

exception $-3 \quad -3$

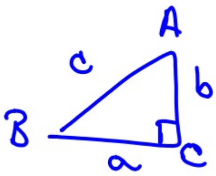
reciprocal $\frac{2}{5} \rightarrow \frac{5}{2}$ $\quad 2x = 4$ $\quad \frac{1}{\sin} = \csc$

quotient $\div$ $\quad \frac{2x}{2} = \frac{4}{2}$

symmetry $x = 2$ specific

trig identities

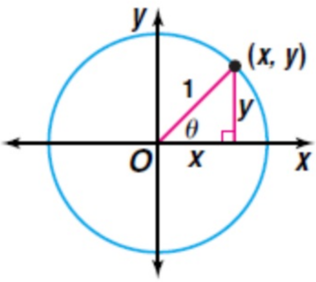
pythagorean $a^2 + b^2 = c^2$



opposite angle

parking lot

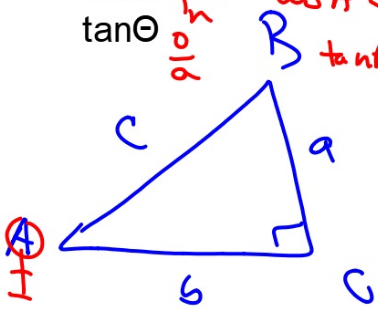
whiteboards (if time)



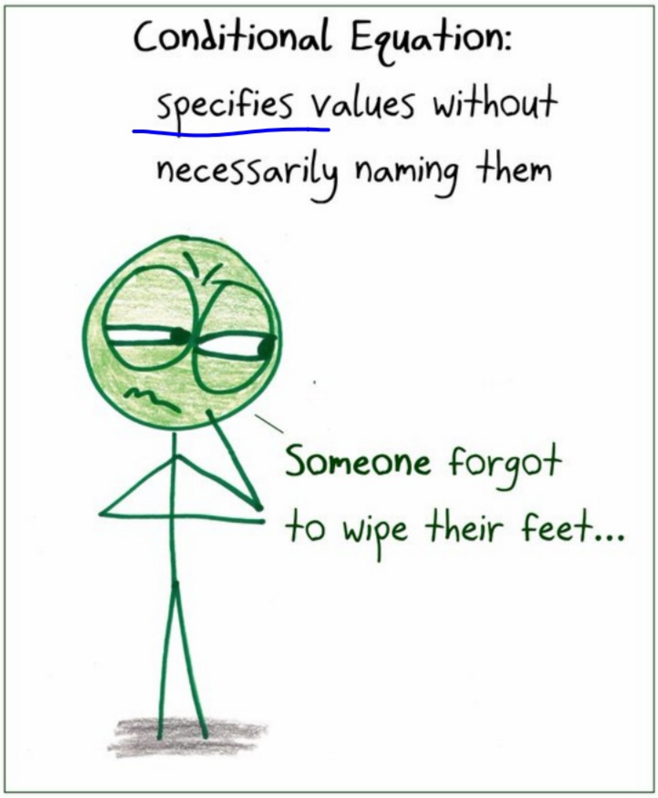
sinΘ $\frac{o}{h}$ $\quad \sin A = \frac{a}{c}$

cosΘ $\frac{a}{h}$ $\quad \cos A = \frac{b}{c}$

tanΘ $\frac{o}{a}$ $\quad \tan A = \frac{a}{b}$




Identity: an equation that applies to all values
T
Anyone who tracks mud on my carpet will pay the price.


Conditional Equation: specifies values without necessarily naming them
Someone forgot to wipe their feet...

$2(x+3) = 10$ $2(x+3) = 2x + 6$

Callback to geometry: All you need is one exception...

1 **Prove that $\sin x \cos x = \tan x$ is *not* a trigonometric identity by producing a counterexample.**

What x makes F ? if x = 30

$$\sin 30 \cdot \cos 30 \stackrel{?}{=} \tan 30$$

$$\frac{1}{2} \cdot \frac{\sqrt{3}}{2} \stackrel{?}{=} \frac{1}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{4} \neq \frac{\sqrt{3}}{3}$$

Definitions: Start parking lot

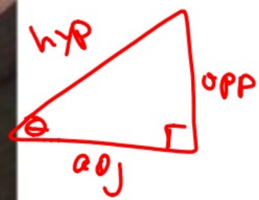
The following trigonometric identities hold for all values of θ where each expression is defined.

$$\sin\theta = \frac{1}{\csc\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

It's not even Summer and I already have a tan



$$\tan\theta = \frac{opp}{adj}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\frac{opp}{adj} = \frac{\frac{opp}{hyp}}{\frac{adj}{hyp}}$$

$$\frac{opp}{adj} = \frac{opp}{adj}$$

The following trigonometric identities hold for all values of θ where each expression is defined.

$$\frac{\sin\theta}{\cos\theta} = \tan\theta \qquad \frac{\cos\theta}{\sin\theta} = \cot\theta$$

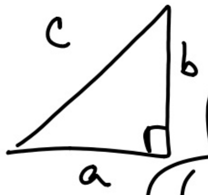
Why? (RT triangle)

$$\frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

$$= \cot\theta$$

Memorize (unit circle)

Pythagorean Identities	The following trigonometric identities hold for all values of θ where each expression is defined. $\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $1 + \cot^2\theta = \csc^2\theta$



$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = \frac{c^2}{c^2}$$

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

$$(\cos x)^2 + (\sin x)^2 = 1$$

derive

$$\left(\frac{\sin^2\theta}{\cos^2\theta}\right) + \left(\frac{\cos^2\theta}{\cos^2\theta}\right) = \left(\frac{1}{\cos^2\theta}\right)$$

$$\tan^2\theta + 1 = \sec^2\theta$$

derive

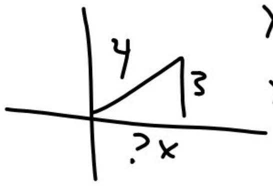
$$\left(\frac{\sin^2\theta}{\sin^2\theta}\right) + \left(\frac{\cos^2\theta}{\sin^2\theta}\right) = \left(\frac{1}{\sin^2\theta}\right)$$

$$1 + (\cot^2\theta) = \csc^2\theta$$

Might be reciprocals
Might need reference triangle

2 **Use the given information to find the trigonometric value.**

a. If sec $\theta = \frac{3}{2}$, find cos θ. $= \frac{2}{3}$



$x^2 + 3^2 = 4^2$

$x^2 + 9 = 16$

$x^2 = 7$

$x = \sqrt{7}$

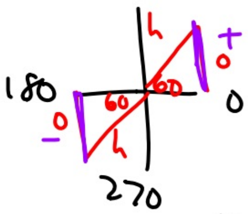
$\frac{1}{\sin}$

b. If csc $\theta = \frac{4}{3}$, find tan θ. $= \frac{opp}{adj} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$

$\sin\theta = \frac{3}{4}$

3 **Express each value as a trigonometric function of an angle in Quadrant I.**

a. $\sin 600^\circ$ $= \sin 240^\circ = -\sin 60^\circ$

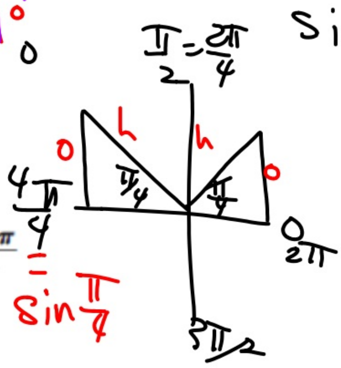


(Compare to Quadrant 1 reference angle)
Uses same reference angle:
is the sign same or opposite?

$\sin \frac{19\pi}{4} - \frac{8\pi}{4}$

$\frac{11\pi}{4} - \frac{8\pi}{4}$

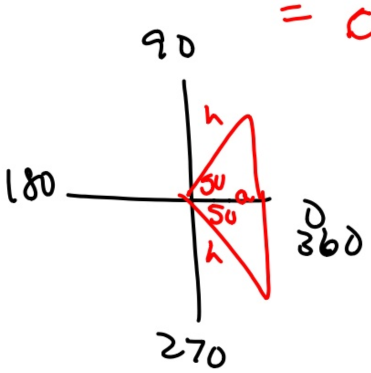
$\sin \frac{3\pi}{4}$



b. $\sin \frac{19\pi}{4}$ $= \sin \frac{\pi}{4}$

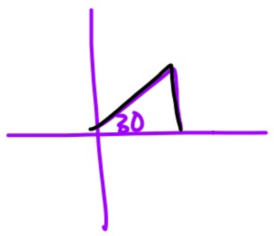
c. $\cos(-410°)$ $= \cos 310$

$= \cos 50$



d. $\tan \frac{37\pi}{6}$ $= \tan 1110 = \tan 30$ -1

$\tan 30 = \tan \frac{\pi}{6}$



Same reference triangles but in different quadrants: what has changed?

same or opposite?

Opposite-Angle Identities	The following trigonometric identities hold for all values of A.
	sinA1 $\sin(-A) = -\sin A$
	cosA $\cos(-A) = \cos A$

CW

CCW

-

What quadrant?
Sides of reference triangle
Answer the question

Use the given information to determine the exact trigonometric value.

8. $\cos\theta = \frac{2}{3}$, $0° < \theta < 90°$; $\sec\theta$

9. $\cot\theta = -\frac{\sqrt{5}}{2}$, $\frac{\pi}{2} < \theta < \pi$; $\tan\theta$

Express each value as a trigonometric function of an angle in Quadrant I.

same sign or opposite?

12. $\cos \frac{7\pi}{3}$

13. $\csc(-330°)$

Try to get to a single trig function: what can you substitute?
(parking lot)

Simplify each expression.

14. $\dfrac{\csc \theta}{\cot \theta}$

4 Simplify $\sin x + \sin x \cot^2 x$.

Rules of algebra apply:
you can factor
combine like terms
you can substitute...
(parking lot)

15. $\cos x \csc x \tan x$

16. $\cos x \cot x + \sin x$