Precalc 10.4

Use and determine standard and general forms for hyperbolas
Graph hyperbolas

hyperbola

focus (foci)

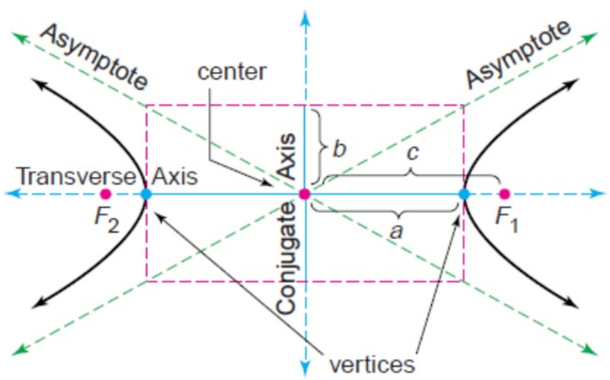
asymptotes

transverse axis

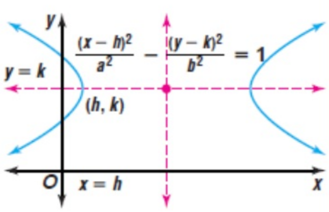
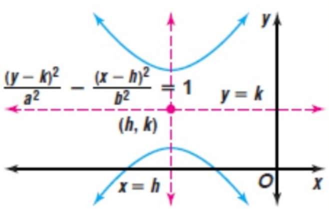
conjugate axis

standard form

eccentricity e=c/a



Note that $c > a$ for the hyperbola.

Standard Form of the Equation of a Hyperbola	Orientation	Description
x $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, where $b^2 = c^2 - a^2$		center: (h, k) foci: $(h \pm c, k)$ vertices: $(h \pm a, k)$ equation of transverse axis: $y = k$ *(parallel to x-axis)*
y $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, where $b^2 = c^2 - a^2$		center: (h, k) foci: $(h, k \pm c)$ vertices: $(h, k \pm a)$ equation of transverse axis: $x = h$ *(parallel to y-axis)*

transverse axis...a
conjugate axis...b

$a^2 + b^2 = c^2$

General form: Complete the square (be careful!)

For the equation of each hyperbola, find the coordinates of the center, the foci, and the vertices and the equations of the asymptotes of its graph. Then graph the equation.

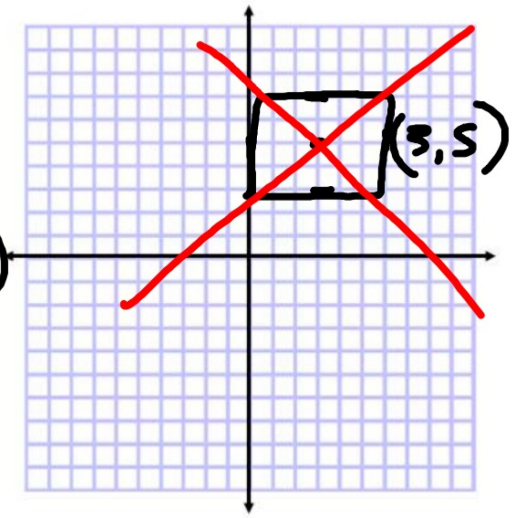
20. $-4x^2 + 9y^2 - 24x - 90y + 153 = 0$

$(-4x^2 - 24x \quad) + (9y^2 - 90y$

$-4(x^2 - 6x + 9) + 9(y^2 - 10y + 25)$

$\frac{-4(x-3)^2}{36} + \frac{9(y-5)^2}{36} = \frac{36}{36}$

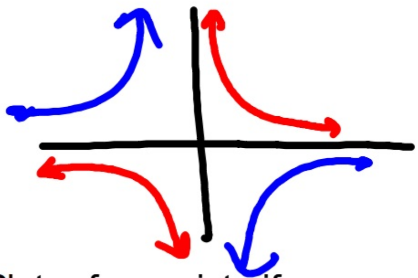
$\frac{(y-5)^2}{4} - \frac{(x-3)^2}{9} = 1$



$y - 5 = \pm\frac{2}{3}(x - 3)$

"tilted hyperbola"

A special case of the equilateral hyperbola is a "**rectangular hyperbola**", where the coordinate axes are the asymptotes. The general equation of a rectangular hyperbola is $xy = c$, where c is a nonzero constant. The sign of the constant c determines the location of the branches of the hyperbola.



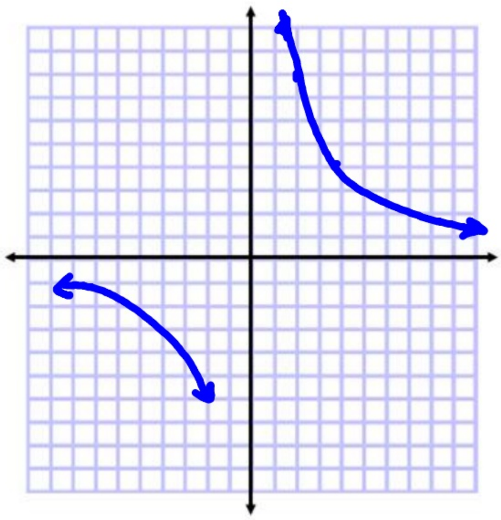
Plot a few points if necessary.

Rectangular Hyperbola: $xy = c$	
Value of c	**Location of branches of hyperbola**
Positive	Quadrants I and III
Negative	Quadrants II and IV

5 Graph $xy = 16$.

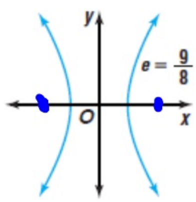
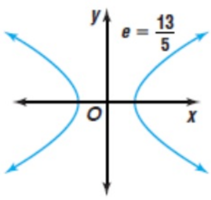
$\frac{xy}{x} = \frac{16}{x}$ $\quad y = \frac{16}{x}$

2	8
4	4



Like an ellipse, the shape of a hyperbola is determined by its eccentricity, which is again defined as $e = \frac{c}{a}$. However, in a hyperbola, $0 < a < c$. So, $0 < 1 < e$ or $e > 1$. The table below shows the relationship between the value of the e and the shape of the hyperbola.

e=c/a (still)

Value of *e*	Graph
close to 1	
not close to 1	

focus is closer to vertex
focus is farther from vertex

e=c/a

6 Write the equation of the hyperbola with center at (3, −1), a focus at (3, −4), and eccentricity $\frac{3}{2}$.

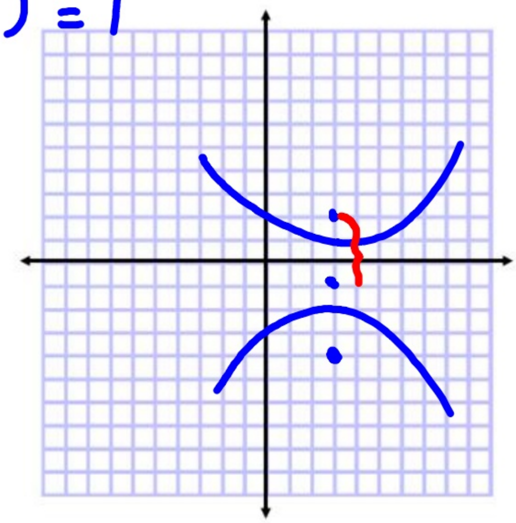
$$e = \frac{3}{2} = \frac{c}{a}$$

$$\frac{3}{2} = \frac{3}{a}$$

$$\frac{3a}{3} = \frac{6}{3}$$

$$a = 2$$

$$\frac{(y+1)^2}{4} - \frac{(x-3)^2}{5} = 1$$



$$4 + b^2 = 3^2$$

$$b^2 = 5$$

$$b = \sqrt{5}$$