Precalc 10.2

Determine and use standard form for the equation of a circle*
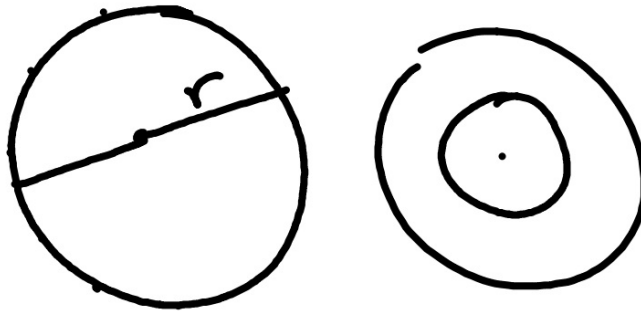Determine and use general form for the equation of a circle*
Graph circles*

circle

radius

center
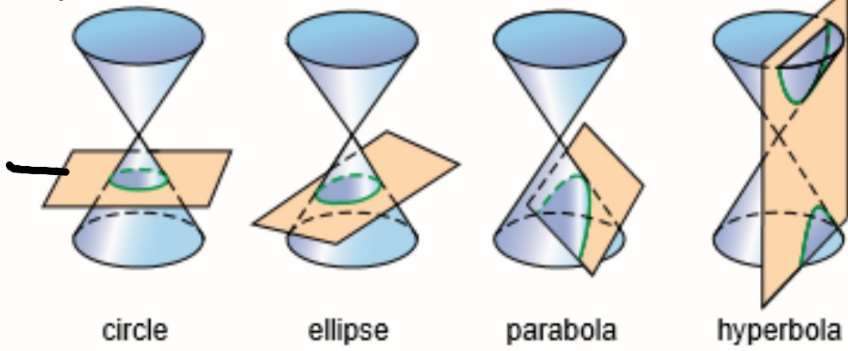
• concentric

conic section

ellipse

parabola

hyperbola

degenerate conic

completing the square

conic models
activity: sidewalk
chalk
   whiteboards

nappe
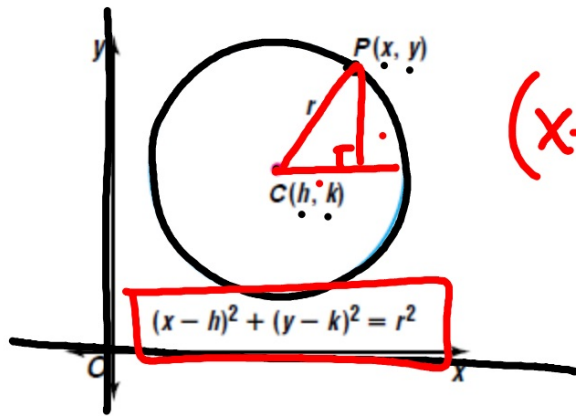

circle   ellipse   parabola   hyperbola

Conic sections

When the plane does pass through the vertex of a conical surface, as illustrated below, the resulting figure is called a **degenerate conic.** A degenerate conic may be a point, line, or two intersecting lines.

point
(degenerate ellipse)

line
(degenerate parabola)

intersecting lines
(degenerate hyperbola)
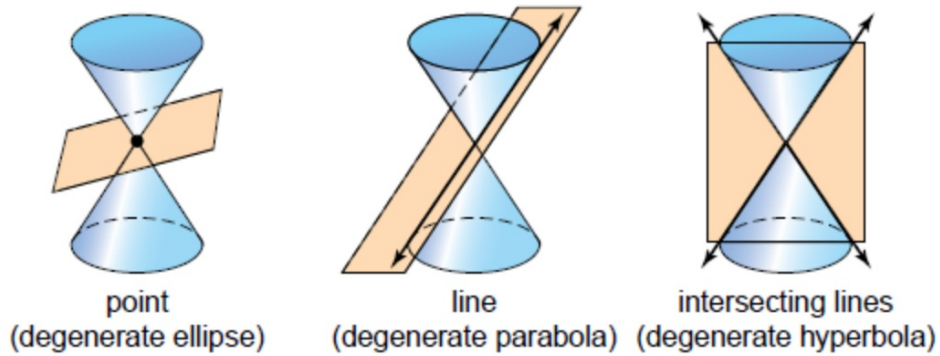
Meh...

Sidewalk chalk

$P(x, y)$

r

$C(h, k)$

$(x - h)^2 + (y - k)^2 = r^2$

$$(x-h)^2 + (y-k)^2 = (r)^2$$

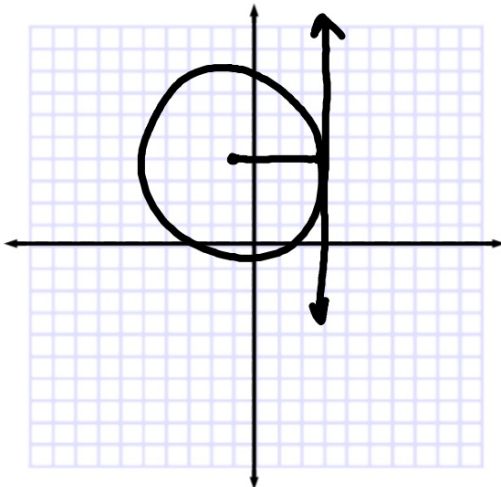| Standard Form of the Equation of a Circle	The standard form of the equation of a circle with radius r and center at (h, k) is $$(x - h)^2 + (y - k)^2 = r^2.$$

$$(\quad)^2 + (\quad)^2 = (\quad)^2$$

Write the standard form of the equation of each circle described. Then graph the equation.

6. center at $(0, 0)$, radius 9

$$(x-0)^2 + (y-0)^2 = 81$$
$$x^2 + y^2 = 81$$

7. center at $(-1, 4)$ and tangent to $x = 3$
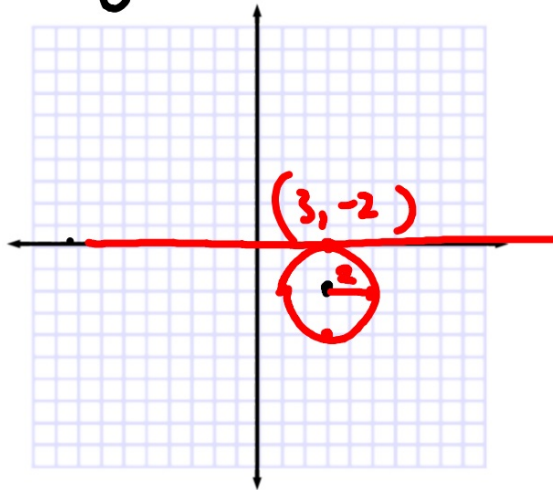
$$(x+1)^2 + (y-4)^2 = 4^2$$
$$= 16$$

What do we need to know to answer the question?
One picture is worth 1000 words...

1 **Write the standard form of the equation of the circle that is tangent to the**
x-axis and has its center at (3, −2). Then graph the equation.

$$(x-3)^2 + (y+2)^2 = 4$$

$(3, -2)$

General Form of the Equation of a Circle	The general form of the equation of a circle is $$x^2 + y^2 + Dx + Ey + F = 0,$$ where D, E, and F are constants.

Multiply out all terms
Arrange in this order

$$(x-3)^2 + (y+2)^2 = 4$$

$$x^2 - 6x + 9 + y^2 + 4y + 4 - 4 = 0$$

$$1x^2 + y^2 - 6x + 4y + 9 = 0$$

$$D = -6 \quad E = 4 \quad F = 9$$

What do we need to know?

$$(x- \ )^2 + (y- \ )^2 =$$

CTS

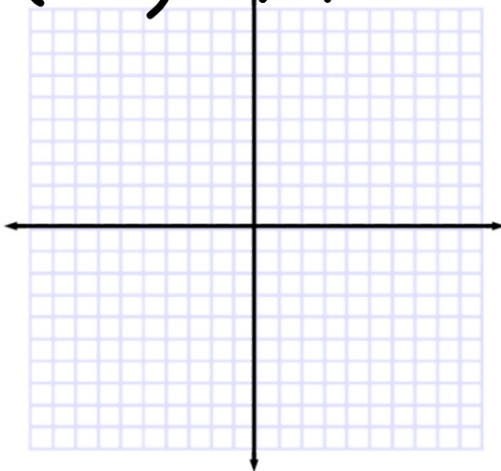**Write the standard form of each equation. Then graph the equation.**

8. $x^2 + y^2 - 4x + 14y - 47 = 0$

9. $\dfrac{2x^2}{2} + \dfrac{2y^2}{2} - \dfrac{20x}{2} + \dfrac{8y}{2} + \dfrac{34}{2} = \dfrac{0}{2}$ (+17)

$(x^2 - 4x + 4) + (y^2 + 14y + 49) = 47 + 4 + 49$

$(x-2)^2 + (y+7)^2 = 100$

$C(2, -7) \quad r = 10$
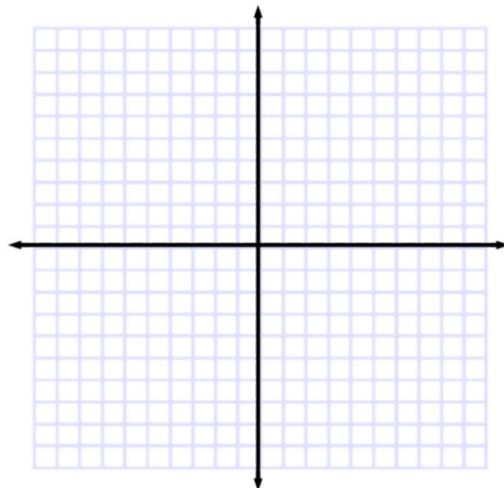

$(x^2 - 10x + 25) + (y^2 + 4y + 4) = -17$
   +25
   +4

$(x-5)^2 + (y+2)^2 = 12$

$C(5, -2) \quad r = \sqrt{12} = 2\sqrt{3}$
                    4·3

What do we need to know?

3 The equation of a circle is $2x^2 + 2y^2 - 4x + 12y - 18 = 0$.

   a. Write the standard form of the equation.

   b. Find the radius and the coordinates of the center.

   c. Graph the equation.

Tough: system of 3 equations
What do we need to know?

$$(x- \quad)^2 + (y- \quad)^2 = (\quad)^2$$

④ Write the standard form of the equation of the circle that passes through the points at (5, 3), (−2, 2), and (−1, −5). Then identify the center and radius of the circle.

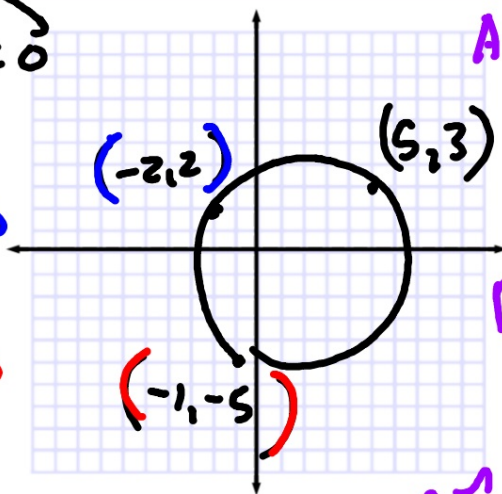| General Form of the Equation of a Circle	The general form of the equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$, where D, E, and F are constants.

$(5,3)$ $25+9+5D+3E+F=0$
$5D+3E+F=-34$

$(-2,2)$ $4+4-2D+2E+F=0$
$-2D+2E+F=-8$

$(-1,-5)$ $1+25-D-5E+F=0$
$-D-5E+F=-26$

$A = \begin{bmatrix} 5 & 3 & 1 \\ -2 & 2 & 1 \\ -1 & -5 & 1 \end{bmatrix}$
3×3

$B = \begin{bmatrix} -34 \\ -8 \\ -26 \end{bmatrix}$
3×1

$A^{-1} \cdot B = \begin{bmatrix} D & -4 \\ E & 2 \\ F & -20 \end{bmatrix}$

Use technology to solve the system...
3x3 · 3x1
(inverse matrix)

$$x^2 + y^2 - 4x + 2y - 20 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 2y + 1) = 20 + 4 + 1$$

$$(x-2)^2 + (y+1)^2 = 25$$

$$C(2,-1) \quad r = 5$$

How can we leverage technology to help?
Matrices! :)