

Precalc 10.7 $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

Find the equations of conic sections that have been rotated or translated

Graph rotations and/or translations of conics

Use the discriminant to identify conic sections

Find the angle of rotation for a given equation

Quiz 10.5-10.6
Thurs.

rotation

Bxy

translation

$$T(-3, 5)$$

$$x^2 + y^2 = 16$$
$$(x+3)^2 + (y-5)^2 = 16$$

discriminant

degenerate case

$T_{(h, k)}$ ↔ translation with respect to (h, k)

replace x with $(x-h)$
replace y with $(y-k)$
(just like moving center of circles...)

Identify the graph of each equation. Write an equation of the translated or rotated graph in general form.

5. $x^2 + y^2 = 7$ for $T_{(3,2)}$

$$(x-3)^2 + (y-2)^2 = 7$$

$$x^2 - 6x + 9 + y^2 - 4y + 4 = 7$$

$$x^2 + y^2 - 6x - 4y + 6 = 0$$

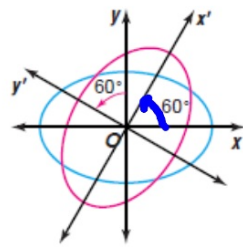
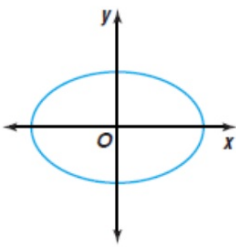
6. $y = 2x^2 - 7x + 5$ for $T_{(-4,5)}$

$$(y-5) = 2(x+4)^2 - 7(x+4) + 5$$

$$x^2 + 8x + 16$$

$$-7 \cdot 5 = 2x^2 + 16x + 32 - 7x - 28 + 5$$

$$2x^2 + 9x - y + 42 = 0$$



60° counterclockwise rotation

default CCW

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

To find the equation of a conic section with respect to a rotation of θ ,
replace

$$\begin{aligned} x &\text{ with } x' \cos \theta + y' \sin \theta \\ \text{and } y &\text{ with } -x' \sin \theta + y' \cos \theta \end{aligned}$$

$\theta = \text{angle}$

Will write on board for test/quiz *but you have to know what to do with it...*

$x \rightarrow x' y'$

2 Find the equation of the graph of $\frac{x^2}{16} - \frac{y^2}{9} = 1$ after it is rotated 45° about the origin. Then sketch the graph and its rotation.

What is it?
Simplify fractions first

$$144 \cdot \frac{(x' \cos 45 + y' \sin 45)^2}{16} - \frac{(-x' \sin 45 + y' \cos 45)^2}{9} = 1$$

x with $x' \cos 45 + y' \sin 45$
and y with $-x' \sin 45 + y' \cos 45$

$$9 \left(\frac{1}{2} x'^2 + x'y' + \frac{1}{2} y'^2 \right) = 16 \left(\frac{1}{2} x'^2 - x'y' + \frac{1}{2} y'^2 \right) = 144$$

$$\frac{9}{2} x'^2 + 9x'y' + \frac{9}{2} y'^2 - 8x'^2 + 16x'y' - 8y'^2 = 144$$

Does it still look like the equation of a hyperbola?

$$-\frac{7}{2} x'^2 - \frac{25}{2} y'^2 + 25x'y' - 144 = 0$$

$$B^2 - 4AC$$

$$-7(x')^2 - 25(y')^2 + 50x'y' - 288 = 0$$

A C B

$$50^2 - 4(-7)(-25) = 2500 - 700 = 1800$$

$$(x' \cos 45 + y' \sin 45)^2$$

$$\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$$

$$\frac{\sqrt{2}}{2}x' + \frac{\sqrt{2}}{2}y'$$

$$\frac{1}{2}(x')^2 + \frac{2}{4}x'y' + \frac{2}{4}(y')^2$$

$$\frac{1}{2}x^2 - xy + \frac{1}{2}y^2$$

$$\frac{1}{2}(x')^2 + 1x'y' + \frac{1}{2}(y')^2$$

$$-x \sin 45 + y \cos 45$$

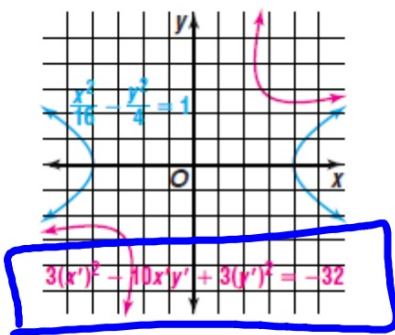
$$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y$$

$$-\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y$$

$$\frac{1}{2}x^2 - \frac{2}{4}xy + \frac{2}{4}y^2$$

* diff prob. !!

The graph below shows the hyperbola and its rotation.



Discriminant (alg 1 & 2)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$13 - 310$$

$$B^2 - 4AC$$

Identifying
Conics By
Using the
Discriminant



For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

- if $B^2 - 4AC < 0$, the graph is a circle ($A = C, B = 0$) or an ellipse ($A \neq C$ or $B \neq 0$);
- if $B^2 - 4AC > 0$, the graph is a hyperbola;
- if $B^2 - 4AC = 0$, the graph is a parabola.

Needed if something has been rotated Bxy...
If B=0 can still identify by inspection

When XY term (rotation) all bets are off..

sine 45, cos 30 etc. changes everything

$b^2 - 4ac$

$$\tan^{-1}\left(\frac{4}{3}\right) = 2\theta$$

$$2\theta = 53$$

3 Identify the graph of the equation $x^2 - 4xy + 4y^2 + 5\sqrt{3}y + 1 = 0$.

A B C

$$\theta = 26.6$$

$$B^2 - 4AC$$

$$(-4)^2 - 4 \cdot 1 \cdot 4$$

$$16 - 16 = 0$$

$$\frac{-4}{1-4} = \frac{-4}{-3} = \frac{4}{3}$$

Angle of
Rotation
About the
Origin

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the angle of rotation θ about the origin can be found by

$$\theta = \frac{\pi}{4}, \text{ if } A = C, \text{ or}$$
$$\tan 2\theta = \frac{B}{A - C}, \text{ if } A \neq C.$$

$$\tan 2\theta = \frac{B}{A - C}$$

=

- 4 Identify the graph of the equation $2x^2 + 9xy + 14y^2 - 5 = 0$. Then find θ and use a graphing calculator to draw the graph.

$$\tan 2\theta = \frac{B}{A-C}$$

Desmos

Graphing calculator: must solve for $y =$
Use quadratic formula
not sure it is worth it..

Desmos?

- 4 Identify the graph of the equation $2x^2 + 9xy + 14y^2 - 5 = 0$. Then find θ and use a graphing calculator to draw the graph.