

Precalc 11.3

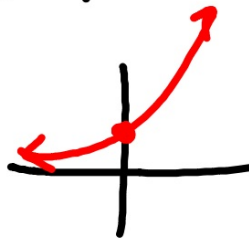
Use the exponential function $y=e^x$ $\approx 2.718 \dots$

Quiz 11.1-11.2 tomorrow

exponential change

growth (incr.)

decay (decrease)



irrational

e

compound interest

continuously compounded interest

activity: Newton's law of cooling

Exponential
Growth or
Decay
(in terms of e)

$N = N_0 e^{kt}$, where N is the final amount, N_0 is the initial amount, k is a constant and t is time.

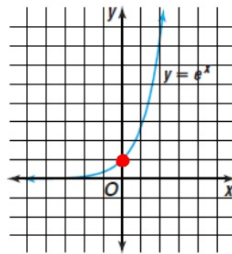
$$N = N_0 e^{kt}$$

How to calculate e

The following computation for e is correct to three decimal places.

$$\begin{aligned} e &= 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \\ &\quad \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} \\ &= 1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} + \frac{1}{720} + \frac{1}{5040} \\ &= 1 + 1 + 0.5 + 0.16667 + 0.04167 + 0.00833 + \\ &\quad 0.00139 + 0.000198 \\ &= 2.718 \end{aligned}$$

The function $y = e^x$ is one of the most important exponential functions. The graph of $y = e^x$ is shown at the right.



Continuously
Compounded
Interest

The equation $A = Pe^{rt}$, where P is the initial amount, A is the final amount, r is the annual interest rate, and t is time in years, is used for calculating interest that is compounded continuously.

Example



2

FINANCE Compare the balance after 25 years of a **\$10,000** investment earning 6.75% interest compounded continuously to the same investment compounded semiannually.

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$
$$= 10,000 \left(1 + \frac{0.0675}{2}\right)^{2 \cdot 25}$$
$$= 52,574.62$$

$$A = 10,000 \left(e^{0.0675(25)} \right)^{1.6875}$$
$$= \$54,059.49$$

9. **Physics** Newton's Law of Cooling expresses the relationship between the temperature of a cooling object and the time t elapsed since cooling began. This relationship is given by $y = ae^{-kt} + c$, where c is the temperature of the medium surrounding the cooling object, a is the difference between the initial temperature of the object and the surrounding temperature, and k is a constant related to the cooling object.

- a. The initial temperature of a liquid is 160°F . When it is removed from the heat, the temperature in the room is 76°F . For this object, $k = 0.23$. Find the temperature of the liquid after 15 minutes.
- b. Alex likes his coffee at a temperature of 135° . If he pours a cup of 170°F coffee in a 72°F room and waits 5 minutes before drinking, will his coffee be too hot or too cold? Explain. For Alex's cup, $k = 0.34$.

$$= 84e^{-0.23 \cdot 15} + 76$$

9-150