* Algebra 2 Ch. 5

Precalc 11.1

Use the properties of exponents*
Evaluate and simplify expressions with rational exponents*
Solve equations containing rational exponents

base

exponent $\sqrt{17}$ $17^{\frac{1}{2}}$

rational

scientific notation $\sqrt[3]{15}$ $15^{\frac{1}{3}}$

radical form vs rational exponent form

activity: operation pathways
graphing calculator

AEROSPACE On July 4, 1997, the Mars Pathfinder Lander touched down on Mars. It had traveled 4.013×10^8 kilometers from Earth. Two days later, the Pathfinder's Sojourner rover was released and transmitted data from Mars until September 27, 1997.

1 **AEROSPACE** **At their closest points, Mars and Earth are approximately 7.5×10^7 kilometers apart.**

a. Write this distance in standard form.

$7.5 \times 10^7 = 7.5\ (10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10)$ or 75,000,000

75,000,000

b. How many times farther is the distance Mars Pathfinder traveled than the minimum distance between Earth and Mars?

Desmos

GRAPHING CALCULATOR EXPLORATION

Recall that if the graphs of two equations coincide, the equations are equivalent.

TRY THESE **Graph each set of equations on the same screen. Use the graphs and tables to determine whether Y_1 is equivalent to Y_2 or Y_3.**

1. $Y_1 = x^2 \cdot x^3$, $Y_2 = x^5$, $Y_3 = x^6$
2. $Y_1 = (x^2)^3$, $Y_2 = x^5$, $Y_3 = x^6$

WHAT DO YOU THINK?

3. Make a conjecture about the value of $a^m \cdot a^n$.
4. Make a conjecture about the value of $(a^m)^n$.
5. Use the graphing calculator to investigate the value of an expression like $\left(\frac{a}{b}\right)^m$. What do you observe?

$(x^2)^3 = x^2 \cdot x^2 \cdot x^2$

p. 691

Properties of Exponents		
Suppose *m* and *n* are positive integers and *a* and *b* are real numbers. Then the following properties hold.		
Property	**Definition**	**Example**
Product	$a^m a^n = a^{m+n}$	$16^3 \cdot 16^7 = 16^{3+7}$ or 16^{10}
Power of a Power	$(a^m)^n = a^{mn}$	$(9^3)^2 = 9^{3 \cdot 2}$ or 9^6
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where $b \neq 0$	$\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$ or $\frac{243}{1024}$
Power of a Product	$(ab)^m = a^m b^m$	$(5x)^3 = 5^3 \cdot x^3$ or $125x^3$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$	$\frac{15^6}{15^2} = 15^{6-2}$ or 15^4

Whiteboards

2 **Evaluate each expression.**

a. $\frac{2^4 \cdot 2^8}{2^5}$ $= 2^7$

$\frac{2^{12}}{2^5}$

b. $\left(\frac{2}{5}\right)^{-1}$ $\frac{5^1}{2^1}$

$\left(\frac{2}{5}\right)^{-2}$ $\frac{5^2}{2^2}$

3 Simplify each expression.

a. $(s^2t^3)^5$

b. $\frac{x^3y}{(x^4)^3}$

$$\frac{x^3y}{x^{12}} = \frac{y}{x^9}$$

$$x^{-9}y$$

Definition of $b^{\frac{1}{n}}$	For any real number $b \geq 0$ and any integer $n > 1$, $b^{\frac{1}{n}} = \sqrt[n]{b}$. This also holds when $b < 0$ and n is odd.

In this chapter, b will be a real number greater than or equal to 0 so that we can avoid complex numbers that occur by taking an even root of a negative number.

$b^{\frac{1}{4}} = \sqrt[4]{b}$

Keepin' it real...

$b^{\frac{1}{3}} = \sqrt[3]{b}$

$\sqrt{-125}$

$\sqrt[3]{-125} = -5$

4 Evaluate each expression.

a. $125^{\frac{1}{3}}$

b. $\sqrt{14} \cdot \sqrt{7}$ $= \sqrt{98} = 7\sqrt{2}$

$\sqrt{7 \cdot 2}\sqrt{7}$

$49 \wedge 2$

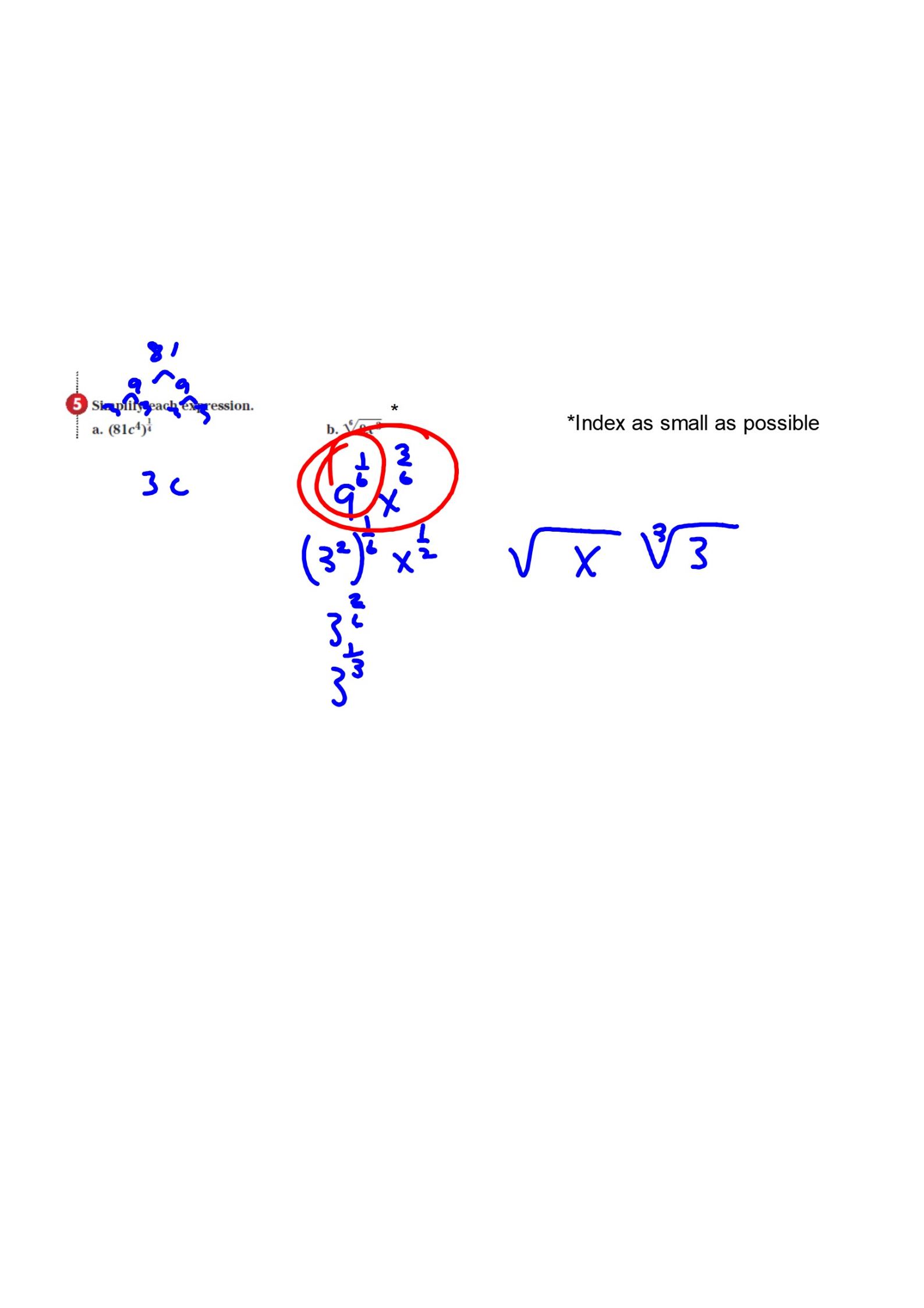
$7 \wedge 7$

5 Simplify each expression.

a. $(81c^4)^{\frac{1}{4}}$

b. $\sqrt[6]{9x^3}$ *

*Index as small as possible



Rational Exponents	For any nonzero number b, and any integers m and n with $n > 1$, and m and n have no common factors $$b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$$ except where $\sqrt[n]{b}$ is not a real number.

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6 Evaluate each expression.

a. $625^{\frac{3}{4}}$

two methods...

1) $(625)^3 = \sqrt[4]{244140625}$

2) $\left(625^{\frac{1}{4}}\right)^3$

625

25 25

5 5 5 5

$5^3 = 125$

b. $\frac{16^{\frac{3}{4}}}{16^{\frac{1}{4}}}$

$\frac{3}{4} - \frac{1}{4}$

16

$16^{\frac{2}{4}} = 16^{\frac{1}{2}} = \sqrt{16}$

7 a. Express $\sqrt[3]{64s^9t^{15}}$ using rational exponents.

$64^{\frac{1}{3}} s^{\frac{9}{3}} t^{\frac{15}{3}} = (2^6)^{\frac{1}{3}} s^3 t^5$

$= 4 s^3 t^5$ *

b. Express $12x^{\frac{2}{3}}y^{\frac{1}{2}}$ using a radical.

$x^{\frac{4}{6}} y^{\frac{3}{6}}$

$12\sqrt[3]{x^2}\sqrt{y}$ $\quad 12\sqrt[6]{x^4 y^3}$ *

8 Simplify $\sqrt{r^7 s^{25} t^3}$.

$r^3 s^{12} t \sqrt{rst}$

9 Solve $734 = x^{\frac{3}{4}} + 5$.

$734 = x^{\frac{3}{4}} + 5$

$-5 \qquad -5$

$729 = x^{\frac{3}{4}}$

$(729)^{\frac{4}{3}} = \left(x^{\frac{3}{4}}\right)^{\frac{4}{3}}$

$6561 = x^1$

21-67.