Quiz 12.1-12.2
Tues.

Precalc 12.3

Find the limit of the terms of an infinite sequence
Find the sum of an infinite geometric series
Use limits to write the fraction form of a repeating decimal

geometric sequence

common ratio

geometric series

finite sequence specific no. of terms

geom

$r<1$ $7, \frac{7}{4}, \frac{7}{16}, \frac{7}{64}, \frac{7}{256}, \ldots$

What is the rule?

$\boxed{r = \frac{1}{4}}$

$a_1 r^{(n-1)}$

$a_{10} = 7\left(\frac{1}{4}\right)^{9} = 0.000026703$

$a_{50} = 7\left(\frac{1}{4}\right)^{49} = 2.2 \times 10^{-29}$

$a_{100} = 7\left(\frac{1}{4}\right)^{99} = 1.74 \times 10^{-59}$

$a_{1000} = 7\left(\frac{1}{4}\right)^{999} = 0$

general term

1 **Estimate the limit of** $\frac{9}{5}, \frac{16}{4}, \frac{65}{27}, \ldots, \frac{7n^2+2}{2n^2+3n}, \ldots$

What is 50th term? 3.3984

100th? 3.448

- 500th? 3.4895
- 1000th? 3.4948

What would you estimate to be the millionth term? 3.5

$$a_{100} = \frac{7(1000)^2+2}{2(1000)^2+3(1000)} \quad \frac{7000c}{2^{\,n}}$$

$$\lim_{n\to\infty} \frac{7n^2+2}{2n^2+3n}$$

$$\lim_{n\to\infty} \frac{\frac{7\cancel{n^2}}{\cancel{n^2}}+\frac{2}{n^2}}{\frac{2\cancel{n^2}}{\cancel{n^2}}+\frac{3n}{n^2}} \quad \frac{7+0}{2+0} = \frac{7}{2}$$

$$\frac{3}{n}$$

Too intimidating, note page number p.776

$\lim\limits_{n\to\infty}$

Theorems for Limits

If the $\lim\limits_{n\to\infty} a_n$ exists, $\lim\limits_{n\to\infty} b_n$ exists, and c is a constant, then the following theorems are true.

Limit of a Sum $\lim\limits_{n\to\infty}(a_n + b_n) = \lim\limits_{n\to\infty} a_n + \lim\limits_{n\to\infty} b_n$

Limit of a Difference $\lim\limits_{n\to\infty}(a_n - b_n) = \lim\limits_{n\to\infty} a_n - \lim\limits_{n\to\infty} b_n$

Limit of a Product $\lim\limits_{n\to\infty} a_n \cdot b_n = \lim\limits_{n\to\infty} a_n \cdot \lim\limits_{n\to\infty} b_n$

Limit of a Quotient $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = \frac{\lim\limits_{n\to\infty} a_n}{\lim\limits_{n\to\infty} b_n}$, where $\lim\limits_{n\to\infty} b_n \neq 0$

Limit of a Constant $\lim\limits_{n\to\infty} c_n = c$, where $c_n = c$ for each n

2 **Find each limit.**

a. $\lim_{n\to\infty} \frac{(1+3n^2)}{n^2} = \lim_{n\to\infty} \frac{\frac{1}{n^2} + \frac{3n^2}{n^2}}{\frac{n^2}{n^2}}$

$\frac{n^2}{n^2} = 1$

$= \lim_{n\to\infty} \frac{1}{n^2} + 3\cancel{\frac{n^2}{n^2}}$

x	$\frac{1}{x^2}$
1	$\frac{1}{1}$
10	$\frac{1}{100}$
1000	$\frac{1}{1{,}000{,}000}$
1,000,000	$\frac{1}{\ }$

$= \lim_{n\to\infty} 0 + 3$

$= 3$

1. Can I put in a number for n and just evaluate?
What would happen if we divide every term by n^2 individually?

b. $\lim_{n\to\infty} \frac{5n^2 + n - 4}{n^2 + 1}$

divide each term by highest degree in denominator

3 Find each limit.

a. $\lim_{n\to\infty} \frac{2+5n+4n^2}{2n}$

$$\frac{\frac{2}{n}+\frac{5\cancel{n}}{\cancel{n}}+\frac{4n^2}{n}}{\frac{2n}{n}} \qquad \frac{5+4n}{2}$$

$$\frac{\frac{2}{n^2}+\frac{5n}{n^2}+\frac{4n^2}{n^2}}{\frac{2n}{n^2}}$$

limit does not exist

Consider the series $\frac{1}{5}+\frac{1}{25}+\frac{1}{125}+\cdots$

$r = \frac{1}{5}$

Sum 1st 100 terms... $= \frac{1}{4}$

$\frac{1}{4}$ Sum of 1st 1000 terms...

$$= \frac{a_1(1-r^n)}{1-r} = \frac{\frac{1}{5}\left(1-\cancel{\underbrace{\tfrac{1}{5}^{100}}_{\times 10^{-70}}}\right)}{1-\frac{1}{5}} =$$

$$\frac{\frac{1}{5}\left(1-\cancel{\frac{1}{5}^{1000}}\right)}{1-\frac{1}{5}} \qquad \frac{\frac{1}{5}\cdot 1}{\frac{4}{5}} \cdot \frac{5}{4} \qquad \frac{\frac{1}{5}(1)}{1-\frac{1}{5}} \qquad \frac{\frac{1}{5}}{\frac{4}{5}} \cdot \frac{5}{4} = \frac{1}{4}$$

What happens to terms as n> ∞?

$0 < r < 1$

infinite

Sum of a series

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_n = \frac{a_1}{1-r}$$

Compare to finite series...
Notice r

$-1 < r < 1$

$|r| < 1$

Sum of an Infinite Geometric Series	The sum S of an infinite geometric series for which $|r| < 1$ is given by $S = \frac{a_1}{1-r}$.

Example 4 Find the sum of the series $21 - 3 + \frac{3}{7} - \cdots$.

Can you? (find r)

$$r = -\frac{1}{7}$$

$$\frac{21}{1 - -\frac{1}{7}} = \frac{21}{1 + \frac{1}{7}} = \frac{21}{\frac{8}{7}} \; \frac{7}{8} = \frac{147}{8}$$

12. 3

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