Precalc 12.2

Find the nth term and geometric means of a sequence

Find the sum of n terms of a geometric series

geometric sequence × rule

* 8 (×2)
8 (^{2})

common ratio (r)

2, 4,...?

* 6 (+)
16 (^{2})

geometric means a_1 - - - - - a_7

geometric series

Geometric Sequence	A geometric sequence is a sequence in which each term after the first, a_1, is the product of the preceding term and the common ratio, r. The terms of the sequence can be represented as follows, where a_1 is

$a_1 \quad a_1 r \quad a_1 r^2 \quad a_1 r^3 \quad a_1 r^4$

3, 6, 12, 24, 48...

$r = 2$

$a_{10} = a_1 r^9$

$a_{99} = a_1 r^{98}$

$a_n = a_1 r^{(n-1)}$

The following sequence is an example of a **geometric sequence.**

10, 2, 0.4, 0.08, 0.016, … *Can you find the next term?*

$\div 5$

$\times \frac{1}{5}$

Think in terms of mult (not division)

$r = -\frac{1}{2}$ $\quad -\frac{1}{8} \quad \frac{1}{16} \quad r \quad -\frac{1}{32}$

1 **Determine the common ratio and find the next three terms in each sequence.**

a. $1, -\frac{1}{2}, \frac{1}{4}, \ldots$ $\quad \dfrac{\frac{1}{4}}{-\frac{1}{2}}$

$$\frac{a_n}{a_{n-1}}$$

$$\frac{a_1 r^{n}}{a_1 r^{n-1}} \quad n-(n-1) \quad r^1$$

$3, 6, 12, 24 \ldots$ $\quad \cdot 2 \quad \cdot 2 \quad \cdot 2$

$r = 2 \quad \frac{6}{3} \quad \frac{12}{6} \quad \frac{24}{12}$

1(?) = -1/2 -1/2(?) = 1/4

b. $\underbrace{r - 1}_{a_1}$, $\underbrace{-3r + 3}_{a_2}$, $\underbrace{9r - 9}_{a_3}$, ...

→ $a_4 = -27r + 27$

$a_5 = 81r - 81$

$a_6 = -243r + 243$

$r = ??$

$r = -3$

$$\frac{9r-9}{-3r+3} \quad \frac{9\cancel{(r-1)}}{-3\cancel{(r-1)}}$$

first term	a_1
second term	$a_2 = a_1 r$
third term	$a_3 = a_1 r^2$
fourth term	$a_4 = a_1 r^3$
fifth term	$a_5 = a_1 r^4$
$\vdots$	$\vdots$
nth term	a_n

The nth Term of a Geometric Sequence	The nth term of a geometric sequence with first term a_1 and common ratio r is given by $\boxed{a_n = a_1 r^{n-1}}$.

Example 2 Find an approximation for the 23rd term in the sequence 256, −179.2, 125.44, …

$a_{23} = a_1 r^{22}$

$= 256(-0.7)^{22}$

0.100

Geometric sequences can represent growth or decay.

- For a common ratio greater than 1, a sequence may model growth. Applications include compound interest, appreciation of property, and population growth. $r > 1$
- For a positive common ratio less than 1, a sequence may model decay. Applications include some radioactive behavior and depreciation.

$r < 1$

4 **Write a sequence that has two geometric means between 48 and −750.**

This sequence will have the form 48, -120, 300, −750.

$a_1 \quad a_2 \quad a_3 \quad a_4$

$a_4 = a_1 r^3$

$-750 = 48r^3$

$-15.625 = r^3$

$-2.5 = r$

A **geometric series** is the indicated sum of the terms of a geometric sequence. The lists below show some examples of geometric sequences and their corresponding series.

Geometric Sequence	**Geometric Series**
3, 9, 27, 81, 243	$3 + 9 + 27 + 81 + 243$
$16, 4, 1, \frac{1}{4}, \frac{1}{16}$	$16 + 4 + 1 + \frac{1}{4} + \frac{1}{16}$
$a_1, a_2, a_3, a_4, \ldots, a_n$	$a_1 + a_2 + a_3 + a_4 + \cdots + a_n$

Sum of a Finite Geometric Series

The sum of the first n terms of a finite geometric series is given by $S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$. $= \dfrac{a_1(1-r^n)}{(1-r)}$

Might be more helpful (later) to use the factored form instead:

Example 5 Find the sum of the first ten terms of the geometric series $16 - 48 + 144 - 432 + \cdots$.

$$\boxed{S_{10} = \frac{a_1(1-r^n)}{1-r}} = \frac{16(1-(-3)^{10})}{1-(-3)}$$

$$= \frac{16(1-(59049))}{4}$$

$$= -236{,}192$$

Lesson 12-1 *(Pages 759–765)*

Find the next four terms in each arithmetic sequence.

1. $7, 3, -1, \ldots$
2. $0.5, -1, -2.5, \ldots$
4. $3, 2.8, 2.6, \ldots$
5. $4x, -x, -6x, \ldots$

2, 4 ...

For Exercises 7–13, assume that each sequence or series is arithmetic.

7. Find the 16th term in the sequence for which $a_1 = 2$ and $d = 5$.

10. Find d for the sequence in which $a_1 = 7$ and $a_{13} = 30$.

12. Find the sum of the first 12 terms in the series $2 + 2.8 + 3.6 + \cdots$.

Lesson 12-2 *(Pages 766–773)*

Determine the common ratio and find the next three terms of each geometric sequ

1. $14, 7, 3.5, \ldots$
2. $-2, 4, -8, \ldots$
3. $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \ldots$
4. $10, -5, 2.5, \ldots$
5. $8, 8\sqrt{2}, 16, \ldots$
6. $a^{10}, a^{8}, a^{6}, \ldots$

For Exercises 7–11, assume that each sequence or series is geometric.

7. Find the sixth term of a sequence whose first term is 9 and common ratio is 2.

8. If $r = 4$ and $a_8 = 100$, what is the first term of the sequence?

10. Write a sequence that has two geometric means between 4 and 256.

11. What is the sum of the first six terms of the series $3 + 9 + 27 + \cdots$?