

Precalc 12.1

discrete math

*Algebra 1 Ch. 4

Find the n th term and arithmetic means of a sequence

Find the sum of n terms of an arithmetic sequence

sequence* *list (rule) 2, 4, 6, 8, ...*

arithmetic sequence* *adding rule*

term *$a_1, a_2, a_3 \dots a_n$*

common difference *$d =$*

recursive formula

explicit formula

Arithmetic Sequence

An arithmetic sequence is a sequence in which each term after the first, a_1 , is equal to the sum of the preceding term and the common difference, d . The terms of the sequence can be represented as follows.

$$a_1 = a_1$$

$$a_2 = a_1 + d$$

$$a_3 = a_1 + 2d$$

$$a_4 = a_1 + 3d$$

$$a_5 = a_1 + 4d$$

⋮

$$a_{10} = a_1 + 9d$$

⋮

$$a_n = a_1 + (n-1)d$$

$$a_1, a_1 + d, a_1 + 2d, \dots$$

↑ ↑ ↑

5

8

11

14

17 ...

} recursive
(start a beginning)

$$a_n = a_{n-1} + d$$

explicit
(subs. in for any n)

① Find the next four terms in the arithmetic sequence $-5, -2, 1, \dots$ $4, 7, 10, 13, 16, 19, 22, 25,$
 $28, 31, 34, 37,$

$$d = 3$$

$$a_n = a_1 + (n-1)d$$

$$a_n = -5 + (n-1) \cdot 3$$

$$a_n = -5 + 3n - 3$$

$$a_n = -8 + 3n$$

$$\begin{aligned} a_n &= -8 + 3(50) \\ &= -8 + 150 \\ &= 142 \end{aligned}$$

first term	a_1	
second term	a_2	
third term	a_3	
fourth term	a_4	
fifth term	a_5	
⋮	⋮	
n th term	a_n	$= a_1 + (n-1)d$

2 Find the 47th term in the arithmetic sequence $-4, -1, 2, 5, \dots$

$$a_n = -4 + (n-1)(3)$$

$$= -4 + 3n - 3$$

$$a_n = -7 + 3n$$

$$a_{47} = -7 + 3(47)$$

$$= -7 + 141$$

$$= 134$$

3 Find the first term in the arithmetic sequence for which $a_{19} = 42$ and $d = -\frac{2}{3}$.

↓
 a_1

$$\begin{aligned} a_{19} &= a_1 + 18d \\ 42 &= a_1 + 18\left(-\frac{2}{3}\right) \\ 42 &= a_1 + -12 \\ +12 & \qquad \qquad +12 \\ \hline 54 &= a_1 \end{aligned}$$

- 4 Write an arithmetic sequence that has five arithmetic means between 4.9 and 2.5.

$$\overset{a_1}{4.9} \quad \underline{4.5} \quad \underline{4.1} \quad \underline{3.7} \quad \underline{3.3} \quad \underline{2.9} \quad \overset{a_7}{2.5}$$

~~mean (average)~~

$$\del{99 + 89 + 93 \approx 93.7}$$

$$a_7 = a_1 + 6d$$

$$2.5 = 4.9 + 6d$$

$$-2.4 = 6d$$

$$-0.4 = d$$

Gauss:



1777-1855

List

Arithmetic Sequence

-9, -3, 3, 9, 15

$3, \frac{5}{2}, 2, \frac{3}{2}, 1, \frac{1}{2}$

$a_1, a_2, a_3, a_4, \dots, a_n$

Sum of list

Arithmetic Series

$-9 + (-3) + 3 + 9 + 15$

$3 + \frac{5}{2} + 2 + \frac{3}{2} + 1 + \frac{1}{2}$

$a_1 + a_2 + a_3 + a_4 + \dots + a_n$

World Series

1, 2, 3, 4, ..., 97, 98, 99, 100

101 + 101

50(101)

5050

Sum of a Finite
Arithmetic
Series

The sum of the first n terms of an arithmetic series is given by

$$S_n = \frac{n}{2}(a_1 + a_n).$$

Example 5 Find the sum of the first 60 terms in the arithmetic series
 $9 + 14 + 19 + \dots + 304.$

36.313

9390

17-370, 45