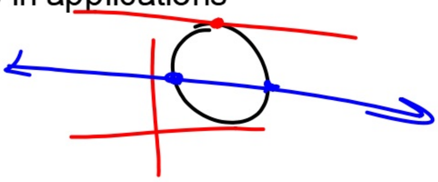
Precalc 15.2

Quiz 15.1 tomorrow

tan line

Find derivatives of polynomial functions
Use derivatives in applications

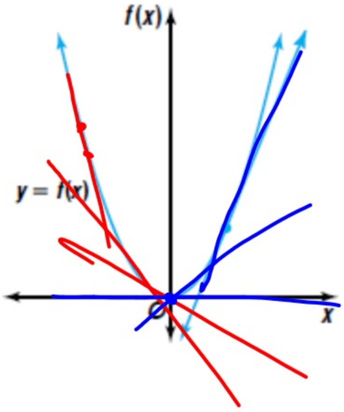


tangent line

secant line

derivative

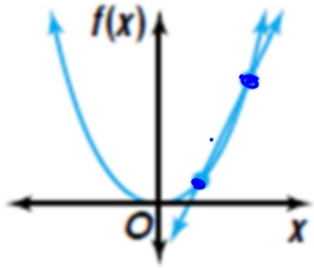
differentiation

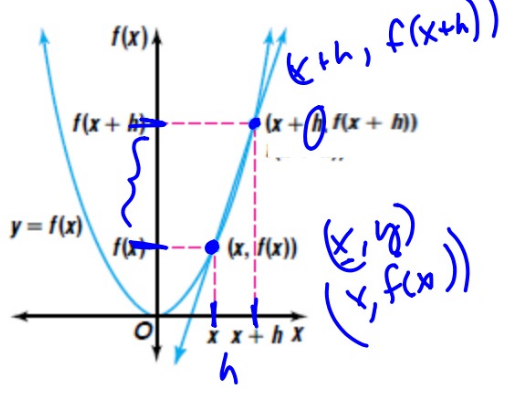


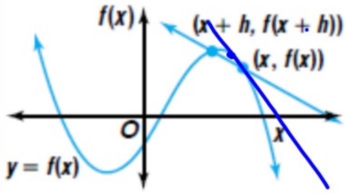
activity: boat and waves

What is slope?

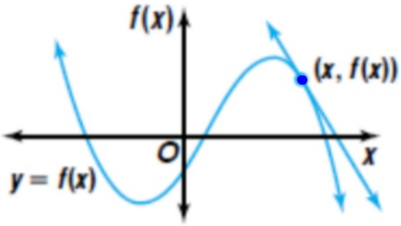
$$m = \frac{\text{rise}}{\text{run}} \quad \frac{\Delta V}{\Delta H}$$


f(x)
O
x


f(x)
f(x + h)
(x + h, f(x + h))
y = f(x)
f(x)
(x, f(x))
O
x
x + h
x

Type of Line	Points of Intersection with Graph	Example	Slope
Secant	2		$m = \dfrac{f(x+h) - f(x)}{h}$

$(x+h)-x$

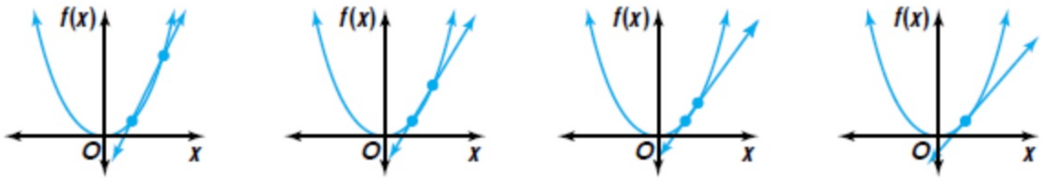
Type of Line	Points of Intersection with Graph	Example	Slope
Tangent	1		$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$y = \sim\sim$

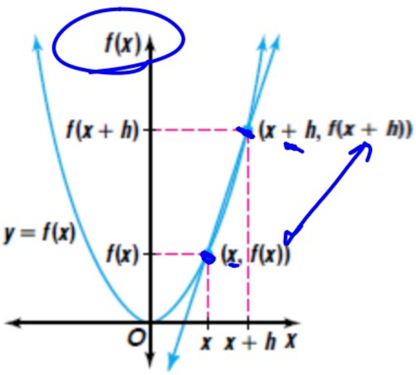
$f(x) = \sim$

$\frac{dy}{dx}$

$f'(x)$


f(x)
O
x
f(x)
O
x
f(x)
O
x
f(x)
O
x

f '(x)



slope (m) =

Derivative of a Function	The derivative of the function $f(x)$ is the function $f'(x)$ given by $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Use the "definition of derivative" to find the derivative of each function.

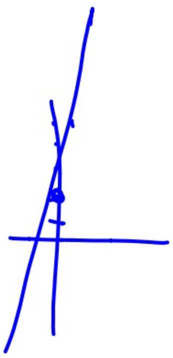
4. $f(x) = 3x + 2$

5. $f(x) = x^2 + x$

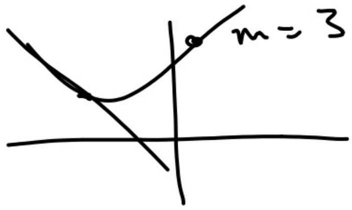
$$f'(x) = \lim_{h \to 0} \frac{(f(x+h)) - (f(x))}{(x+h) - (x)} = \lim_{x \to 0} \frac{3(x+h)+2 - (3x+2)}{h}$$

$$\lim_{h \to 0} \frac{3x + 3h + 2 - 3x - 2}{h}$$

$$\lim_{h \to 0} \frac{3h}{h} = 3$$



5. $f(x) = x^2 + x$



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + (x+h) - (x^2 + x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{x^2 + 2hx + h^2 + x + h - x^2 - x}{h} = \lim_{h \to 0} \frac{2hx + h^2 + h}{h}$$

$$\lim_{h \to 0} \frac{h(2x + h + 1)}{h} = 2x + 0 + 1$$

$$= 2x + 1$$

$2 \cdot 1 + 1$

$2 \cdot -2 + 1$

$-4 + 1$

definition of derivative

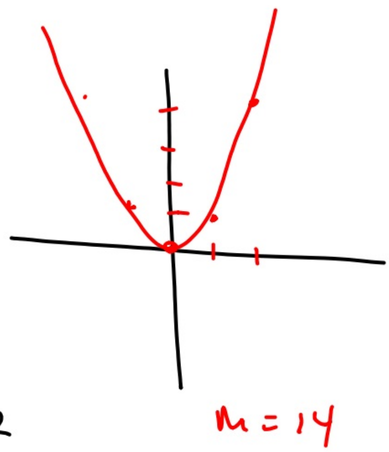
$f'(x)$

$y=x^2$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\left(f(x+h)\right) - \left(f(x)\right)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = 2x + 0 = 2x$$



1 a. Find an expression for the slope of the tangent line to the graph of $y = x^2 - 4x + 2$ at any point. That is, compute $\frac{dy}{dx}$.

b. Find the slopes of the tangent lines when $x = 0$ and $x = 3$.

@

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - fx}{h} = \frac{\left((x+h)^2 - 4(x+h) + 2\right) - \left(x^2 - 4x + 2\right)}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - 4x - 4h + 2 - x^2 + 4x - 2}{h}$$

$$\lim_{h \to 0} \frac{h^2 + 2hx - 4h}{h} = \frac{h(h + 2x - 4)}{h} = 2x - 4$$

$$\frac{dy}{dx} = 2x - 4$$

b) if $x = 0$ $m = -4$
if $x = 3$ $m = 2$

(What's the rule?)

Derivative Rules	Constant Rule:	The derivative of a constant function is zero. If $f(x) = c$, then $f'(x) = 0$.
	Power Rule:	If $f(x) = x^n$, where n is a rational number, then $f'(x) = nx^{n-1}$.
	Constant Multiple of a Power Rule:	If $f(x) = cx^n$, where c is a constant and n is a rational number, then $f'(x) = cnx^{n-1}$.
	Sum and Difference Rule:	If $f(x) = g(x) \pm h(x)$, *then* $f'(x) = g'(x) \pm h'(x)$.

2 **Find the derivative of each function.**

a. $f(x) = x^6$

b. $f(x) = x^2 - 4x + 2$

c. $f(x) = 2x^4 - 7x^3 + 12x^2 - 8x - 10$

Use the derivative rules to find the derivative of each function.

6. $f(x) = 2x^2 - 3x + 5$

7. $f(x) = -x^3 - 2x^2 + 3x + 6$

d. $f(x) = x^3\,(x^2 + 5)$

e. $f(x) = (x^2 + 4)^2$

Use the definition of derivative vs. Use the derivative rules