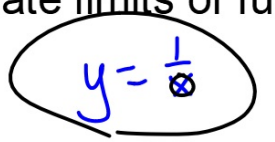
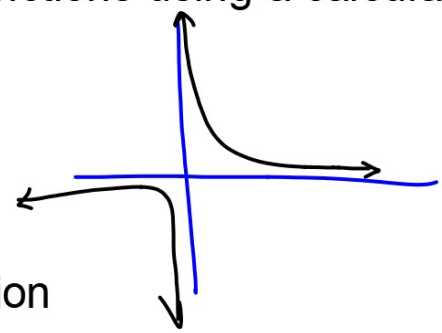
Precalc15.1

Calculate limits of polynomial and rational functions
algebraically
Evaluate limits of functions using a calculator

limit





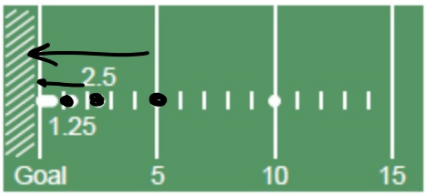
* continuous function

* discontinuous function

Angles are always in radians for calculus!

activity: cooling curve
graphing calculators

SPORTS In football, if the length of a penalty exceeds half the distance to the offending team's goal line, then the ball is moved only half the distance to the goal line. Suppose one team has the ball at the other team's 10-yard line. The other team, in an effort to prevent a touchdown, repeatedly commits penalties. After the first penalty, the ball would be moved to the 5-yard line.



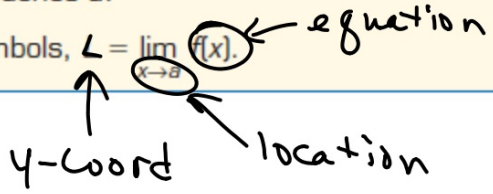
$y = a\left(\frac{1}{2}\right)^x$

Can the ball ever cross the goal line by a penalty?

Limit of a Function

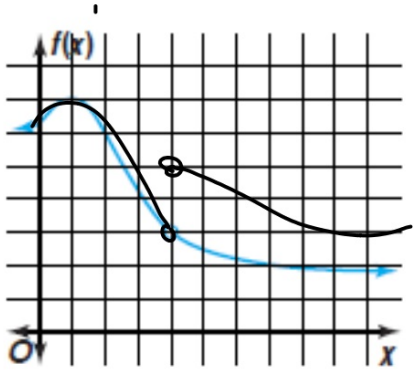
If there is a number L such that the value of $f(x)$ gets closer and closer to L as x gets closer to a number a, then L is called the limit of $f(x)$ as x approaches a.

In symbols, $L = \lim_{x \to a} f(x)$.



The limit is a y-coordinate...

1 **Consider the graph of the function $y = f(x)$ shown at the right. Find each pair of values.**



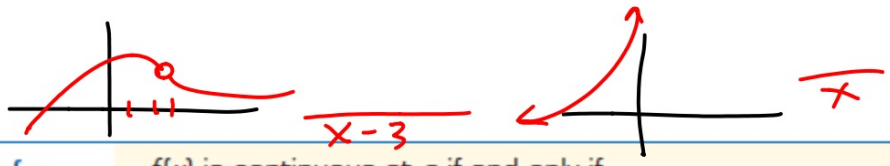
a. $f(2)$ and $\lim_{x \to 2} f(x)$ (Contin.)

6 6

b. $f(4)$ and $\lim_{x \to 4} f(x)$ (discon.)

undef 3

Is it continuous?



Limit of a Continuous Function	$f(x)$ is continuous at a if and only if $\lim_{x \to a} f(x) = f(a)$.

if limit = ordered pair

Examples of continuous functions include polynomials as well as the functions $\sin x$, $\cos x$, and a^x. Also, $\log_a x$ is continuous if $x > 0$.

Example 2 Evaluate each limit.

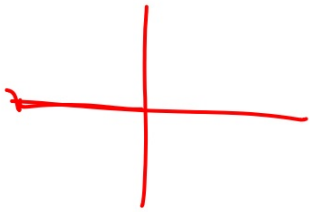
a. $\lim_{x \to 3} (x^3 - 5x^2 + 7x - 10)$ $= -7$

$27 - 45 + 21 - 10$

Try:
direct substitution
→ factor/cancel
graphing calc

reminder: default angles in radians

b. $\lim_{x \to \pi} \frac{\cos x}{x}$ $\frac{\cos\pi}{\pi} = \frac{-1}{\pi}$



-8

-4×2

-2

4 **Evaluate each limit.**

a. $\lim_{x \to 4} \dfrac{x^2 - 2x - 8}{x^2 - 4x}$

$\dfrac{16 - 8 - 8}{16 - 16} = \dfrac{0}{0}$!!

$\lim_{x \to 4} \dfrac{(x-4)(x+2)}{x(x-4)} = \dfrac{6}{4} = \dfrac{3}{2} = 1.5$

b. $\lim_{h \to 0} \frac{h^3 - 4h^2 - 6h}{h}$ $\frac{0-0-0}{0}$:(

$\lim_{h \to 0} \frac{h(h^2 - 4h - 6)}{h}$ $0 - 0 - 6 = -6$

5 **Evaluate each limit.**

a. $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$ **(x is in radians.)** $= \frac{1}{2}$

calculator
(table)

$$\frac{1-\cos 0}{0^2} \quad \frac{1}{\frown}$$

$e \approx 2.7$

b. $\lim_{x \to 1} \left(\frac{\ln x}{x - 1} \right) = 1$

calculator
(table)