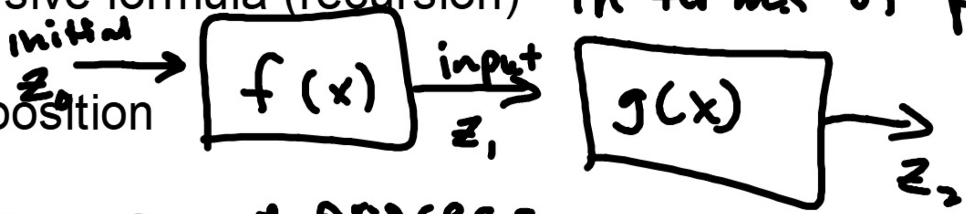
Precalc 12.8

$\ln(-1)$ [illegible] $+ i\pi$

Iterate functions using real and complex numbers

recursive formula (recursion) in terms of previous

composition



iteration – repeat process

$g(f(x))$ $g \circ f(x)$

initial value z_0 not naught

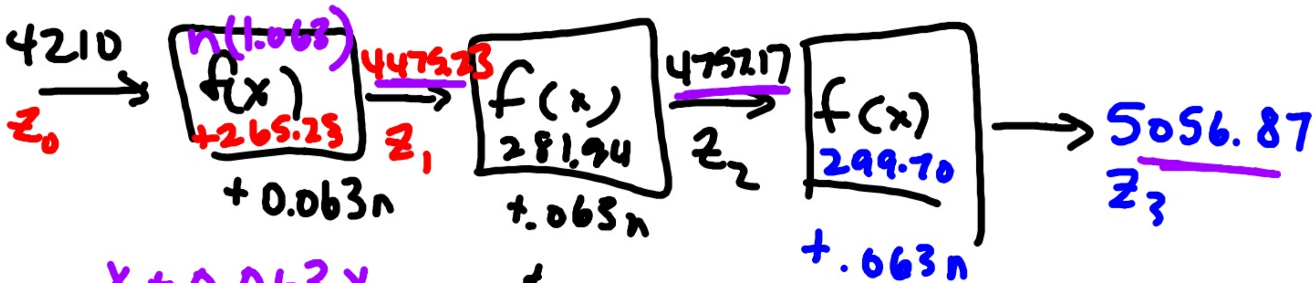
orbit graph 1st n iterates

fractal geometry

prisoner point (set)

gof fof

2 BANKING Selina Anthony has a savings account that has an annual yield of 6.3%. Find the balance of the account after each of the first three years if her initial balance is $4210.



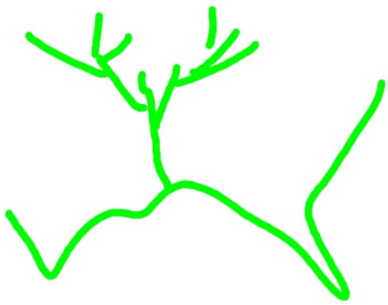
x + 0.063x

1.063x

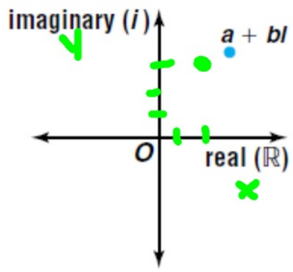
z_1 = 4475.23

z_2 = 4757.17

z_3 = 5056.87



2+3i



Functions can be iterated using the complex numbers as the domain.

Composition functions: f(g(x))...

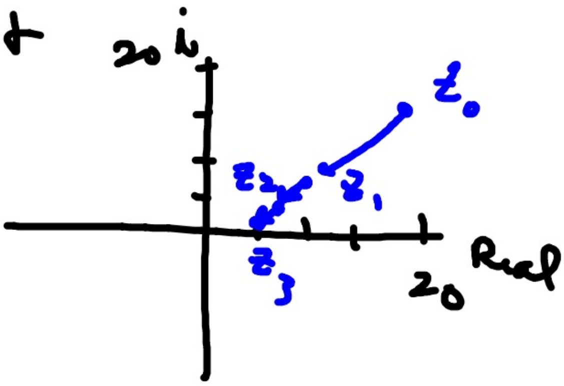
f(z) is the rule

3 Find the first three iterates of the function $f(z) = 0.5z + 2$, if the initial value is $20 + 16i$.

start

$20+16i \rightarrow$ z_0 → [$0.5(20+16i)+2$; $10+8i+2$] → $12+8i$ (z_1) → [$0.5(12+8i)+2$; $6+4i+2$] → $8+4i$ (z_2) → [$0.5(8+4i)+2$; $4+2i+2$] → z_3 $6+2i$

b. Orbit



graph

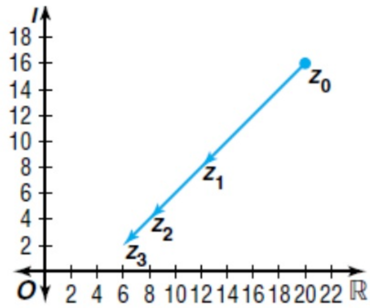
$z_1 = 12+8i$

$z_2 = 8+4i$

$z_3 = 6+2i$

We can graph this sequence of iterates on the complex plane. The graph at the right shows the **orbit,** or sequence of successive iterates, of the initial value of $z_0 = 20 + 16\boldsymbol{i}$ from Example 3.

mx + B

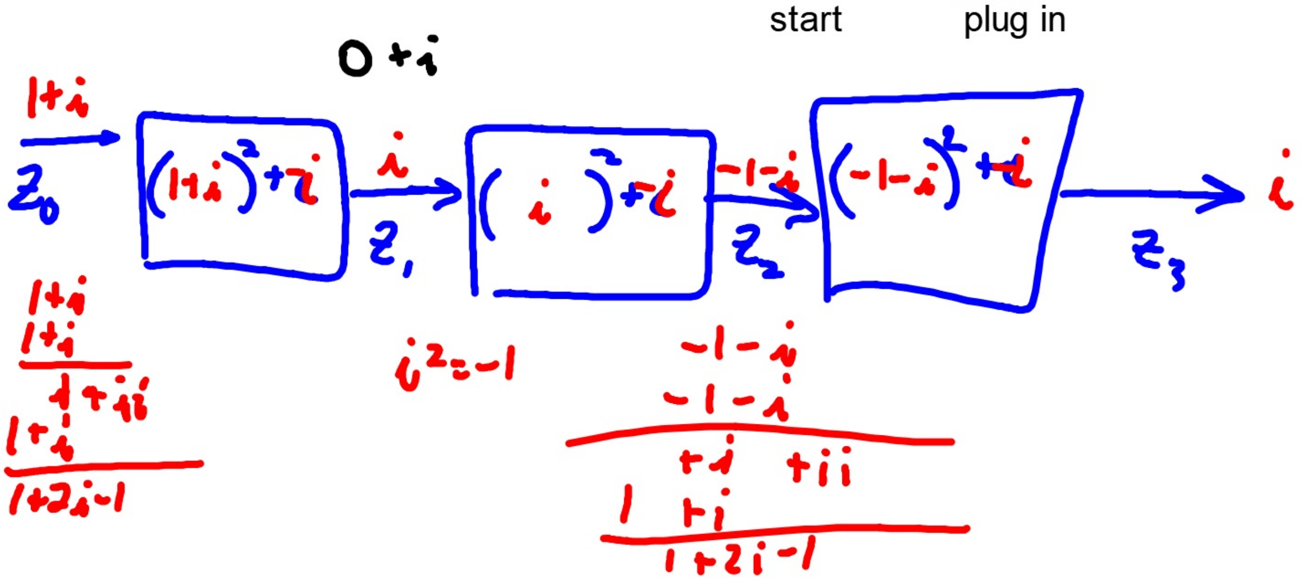


f(z) is the rule

4 Consider the function $f(z) = z^2 + c$, where c and z are complex numbers.

a. Find the first six iterates of the function for $z_0 = 1 + i$ and $c = -i$.

start plug in

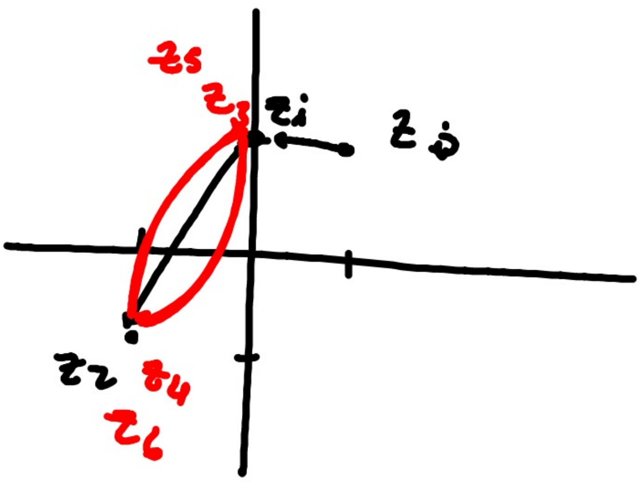


3 max

b. Plot the orbit of the initial point at $1 + i$ under iteration of the function $f(z) = z^2 - i$ for six iterations.

c. Describe the long-term behavior of the function under iteration.

p 817

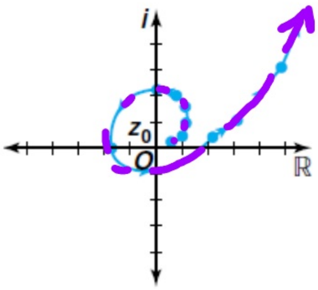


$f(z) = z^2 + c,$

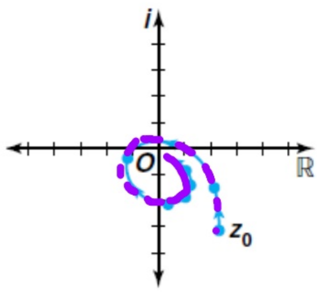
Fractal geometry

prisoner point

escaping point

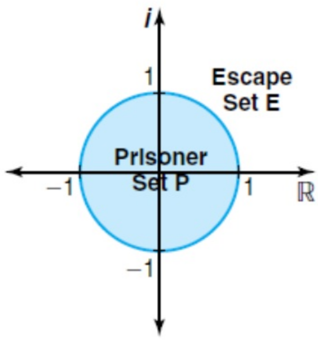


Escaping point

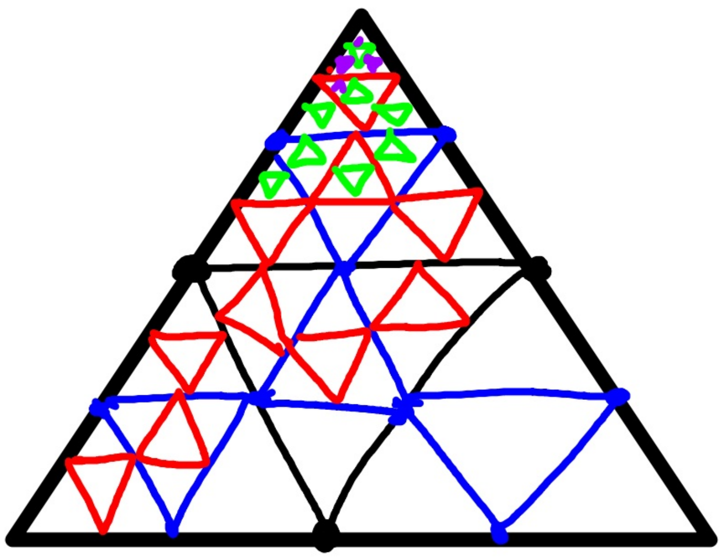


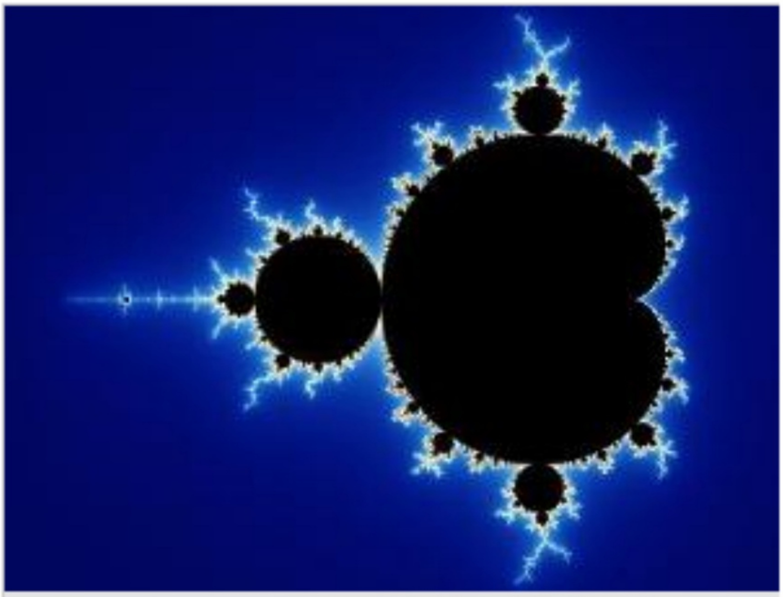
Prisoner point

All of the initial points for a function on the complex plane are split into three sets, those that escape (called the *escape set* E), those that do not escape (called the *prisoner set* P) and the boundary between the escape set and the prisoner set (the Julia set). The escape set, the prisoner set, and the Julia set for the function in Example 5 are graphed at the right. The Julia set in Example 5 is the unit circle.



Equilateral Triangle (iterations)

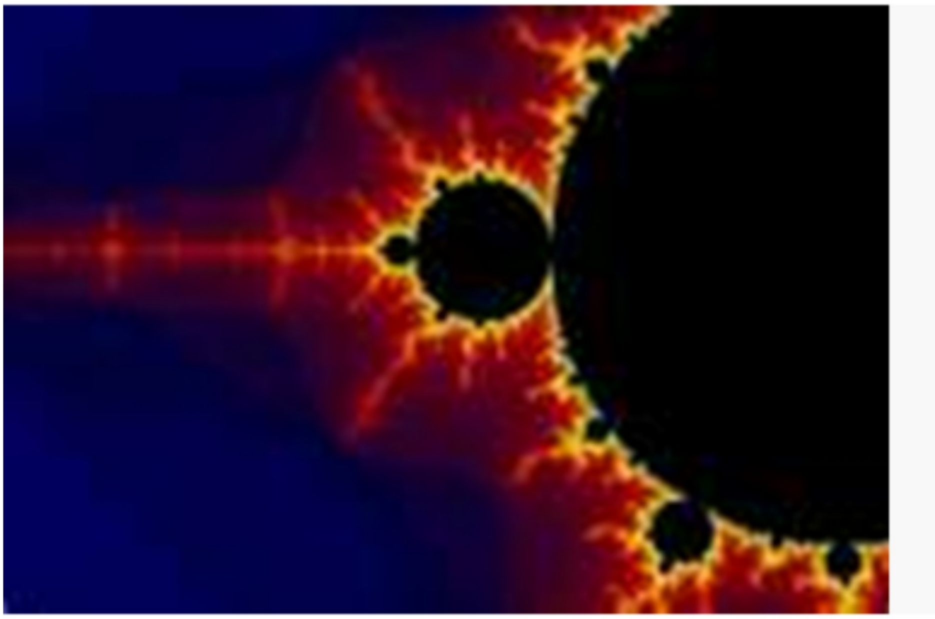




fractal geometry. (Image by Wolfgangbeyer, GFDL)

Mandelbrot set

http://en.wikipedia.org/wiki/Mandelbrot_set

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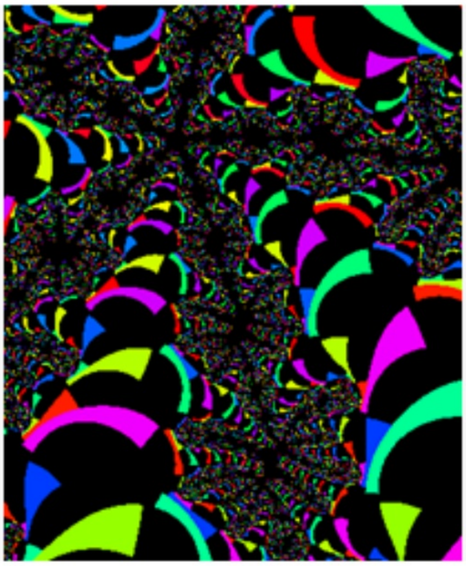

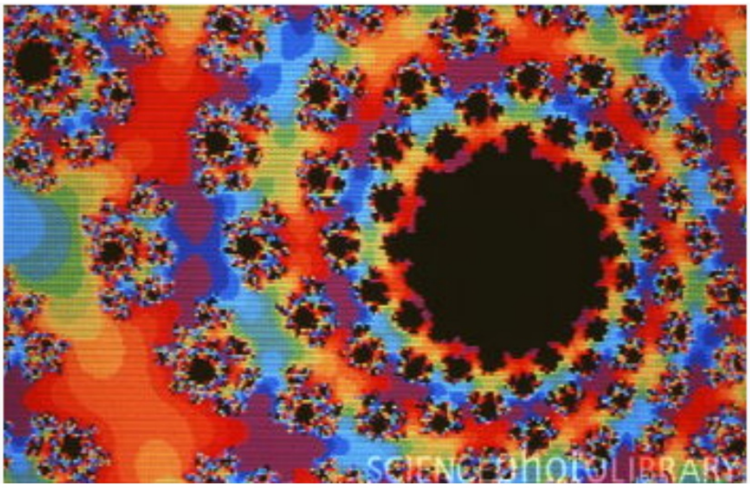

http://www.bing.com/images/search?q=Fractals+In+Nature&FORM=IQFRDR

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35-44

max (3)