Precalc 12.7

Approximate e^x, trig values, and logs by using series
Use Euler's formula to write the exponential form of a complex number

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

Pascal's Triangle

Fibonacci sequence

exponential series

trigonometric series (radians)

Euler's formula

activity: sunflowers & pinecones

Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, 35 . . .

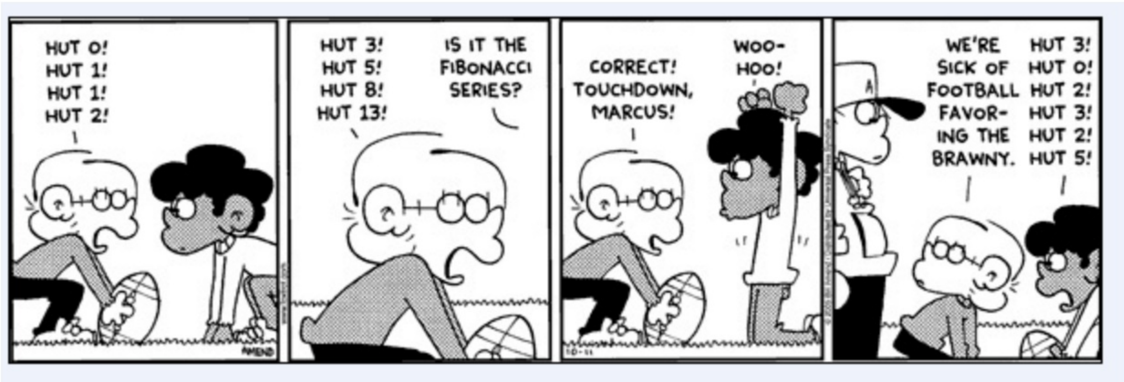
1, 1

Fibonacci in Nature

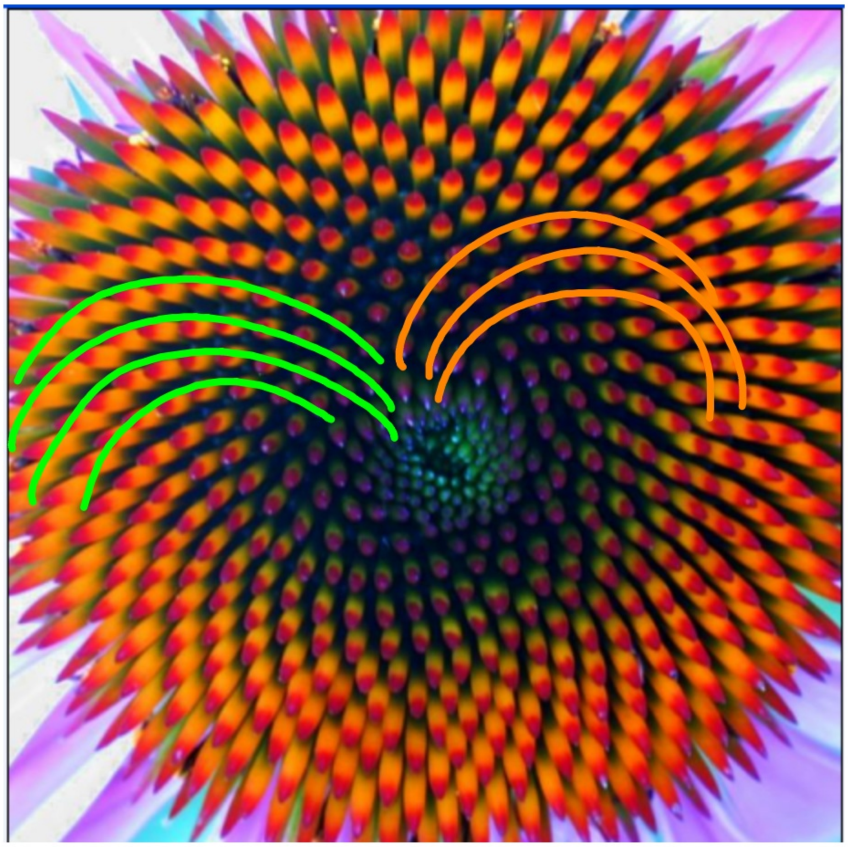
The number of petals on a flower is often a Fibonacci number.

1 3 5 13

Why are four-leaf clovers so rare? Because 4 isn't a Fibonacci number!

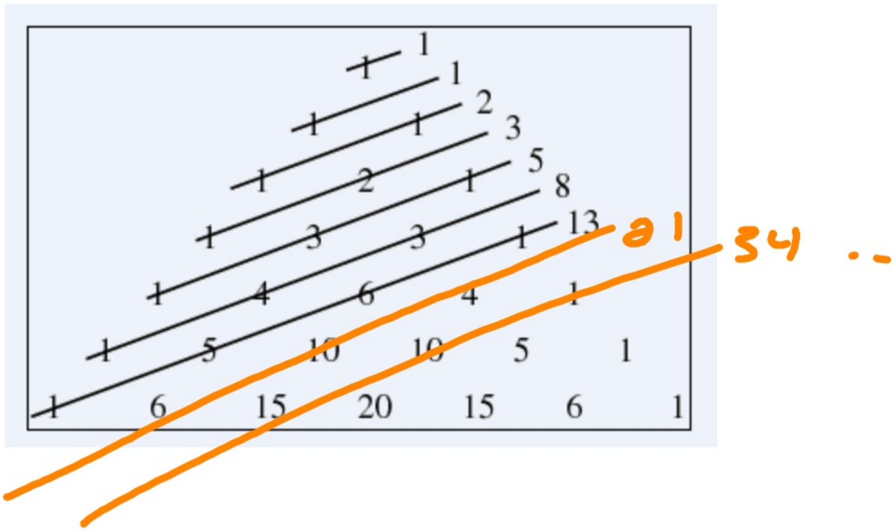


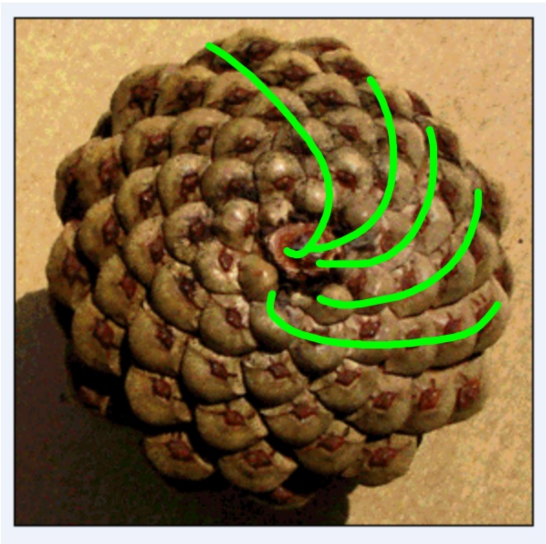
perrin number

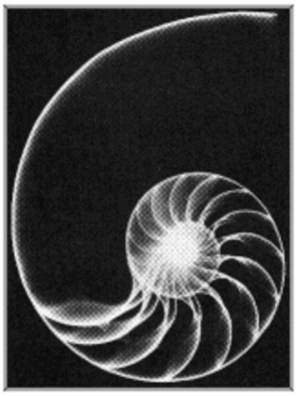


Pascals triangle
What are the sums of diagonals?

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1


1
1
2
3
5
8
13
21
34 ..
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1



definition $0! = 1$

Exponential Series

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$

0 1 2 3

Example 2 Use the first five terms of the exponential series and a calculator to approximate the value of $e^{2.03}$ to the nearest hundredth.

$$e^{2.03} = 1 + 2.03 + \frac{2.03^2}{2} + \frac{2.03^3}{3\cdot2\cdot1} + \frac{2.03^4}{4\cdot3\cdot2\cdot1} + \frac{2.03^4}{120}$$

$$= 1 + 2.03 + 2.060 + 1.394 + 0.7076$$

$$= 6.55476 \approx 6.55 \quad +0.142$$

$$7.614$$

This is why your calculator has to "think" when you use e in calculations.

radians...

Trigonometric Series

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$$

This is why your calculator has to "think about it" when you ask for sin, cos, etc.

3 **Use the first five terms of the trigonometric series to approximate the value of $\cos \frac{\pi}{3}$ to four decimal places.**

cis

Euler's Formula	$e^{i\alpha} = \cos\alpha + i\sin\alpha$

angle in radians
change to polar form

$e^{i\Theta}=1\text{cis}\Theta$

$$a + bi = r(\cos\theta + i\sin\theta)$$
$$= re^{i\theta}$$

ple **4** **Write $1 + \sqrt{3}i$ in exponential form.**

Not real!

$1e^{i\theta} = 1\text{cis}\theta$

$2\text{cis}60^\circ$

$2\text{cis}\frac{\pi}{3} = 2e^{\frac{\pi}{3}i}$

$e^{i\theta} = 1\text{cis}\theta$

Can there be ln of a negative number?

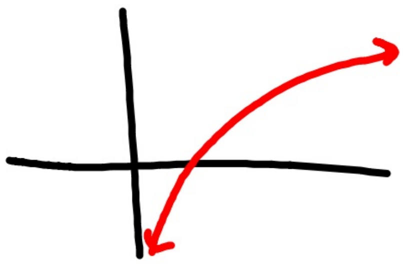
$\ln_e(-) = x$

$e^x = \underline{\qquad}$

$\ln_{2.7}(\quad) =$

just checking...

$2.7^x =$



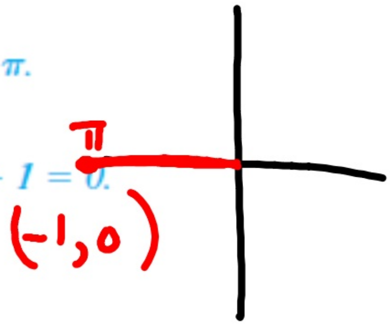
$e^{i\theta} = 1\,\text{cis}\,\theta$

$e^{i\alpha} = \cos\alpha + i\sin\alpha$

$e^{i\pi} = 1\cos\pi + 1\,i\sin\pi$ *Let $\alpha = \pi$.*

$e^{i\pi} = -1 + i(0)$

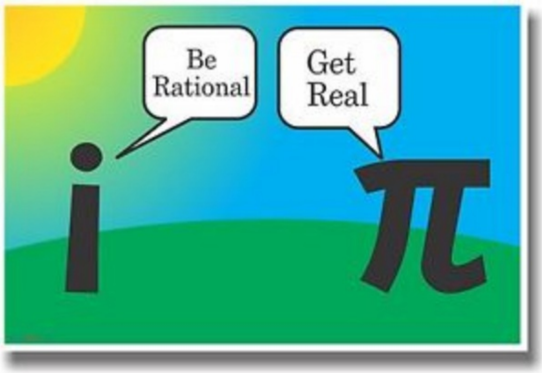
$e^{i\pi} = -1 + 0$ *So $e^{i\pi} + 1 = 0$.*

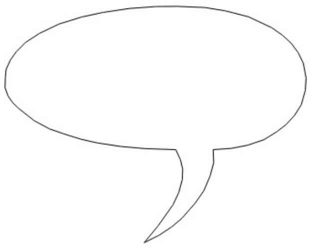


$e^{i\pi} = -1$

$e^{i\pi} + 1 = 0$

Wow, 3 of the most important numbers in one expression! (and also e & i)





-e

$e^{i\pi} = -1$

$e^{i\pi} = -1$ $\ln e^{i\pi} = -1$

$\ln e^{i\pi} = \ln(-1)$

$i\pi = \ln(-1)$

$i\pi(\ln e) = \ln -1$

$i\pi \cdot 1 = \ln -1$

5 **Evaluate $\ln(-270)$.**

$\ln(-270) = \ln(270 \cdot -1)$

$\ln 270 + \ln(-1)$

$\ln 270 + i\pi$

$+i\pi$

$\ln_e(-8)$

$\ln(-1 \cdot 8)$

$\ln 8 + \ln(-1)$ $i\pi$

$\ln 8 + i\pi$

13-4/odd