

Precalc13.5

Find the probability of an event, given the occurrence of another event

conditional probability

sample space

subset

$$P(A|B)$$

Reduced sample space...two-way table

12. **Medicine** To test the effectiveness of a new vaccine, researchers gave 100 volunteers the conventional treatment and gave 100 other volunteers the new vaccine. The results are shown in the table below.

- a. What is the probability that the disease is prevented in a volunteer chosen at random?
- b. What is the probability that the disease is prevented in a volunteer who was given the new vaccine?
- c. What is the probability that the disease is prevented in a volunteer who was not given the new vaccine?

Treatment	Disease Prevented	Disease Not Prevented
New Vaccine	68	32
Conventional Treatment	62	38

Handwritten notes and calculations:

- For question a: $\frac{130}{200} = 65\%$ (circled in blue)
- For question b: $\frac{68}{100} = 68\%$ (circled in blue)
- For question c: $\frac{62}{100} = 62\%$ (circled in blue)
- Row totals: 100 (under New Vaccine), 100 (under Conventional Treatment)
- Column totals: 130 (under Disease Prevented), 70 (under Disease Not Prevented)
- Grand total: 200 (written vertically on the right)

Is the new vaccine better?

**Conditional
Probability**

The conditional probability of event A , given event B , is defined as

$$P(A | B) = \frac{P(A \text{ and } B)}{P(B)} \text{ where } P(B) \neq 0.$$

Find each probability. *die*

4. Two number cubes are tossed. Find the probability that the numbers showing on the cubes match given that their sum is greater than five.

double

$$P(\text{doubles} | \text{sum} > 5)$$

$$\frac{4}{26} = \frac{2}{13}$$

$$\frac{4}{36}$$

$$\frac{26}{36}$$

$$\frac{26}{36}$$

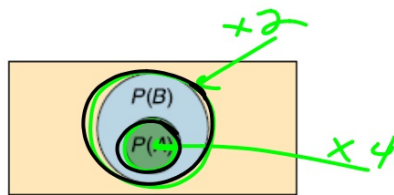
$$\frac{4}{26}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Given: their sum > 5 has already happened...
(How likely is that?)

Use a table...reduced sample space

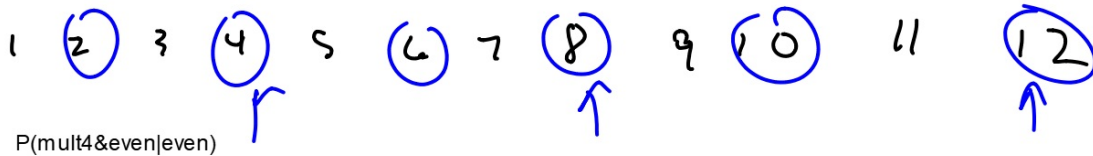
In some situations, event A is a subset of event B . When this occurs, the probability that both event A and event B , $P(A \text{ and } B)$, occur is the same as the probability of event A occurring. Thus, in these situations $P(A|B) = \frac{P(A)}{P(B)}$.



Event A is a subset of event B .

- 4 A 12-sided dodecahedron has the numerals 1 through 12 on its faces. The die is rolled once, and the number on the top face is recorded. What is the probability that the number is a multiple of 4 if it is known that it is even?

$$\frac{3}{6} = \frac{1}{2}$$



A pair of number cubes is thrown. Find each probability given that their sum is greater than or equal to 9.

9. $P(\text{numbers match})$

$$P(\text{doubles} \geq 9)$$

10. $P(\text{sum is even})$

11. $P(\text{numbers match or sum is even})$

$$P(\text{doubles or even} \mid \geq 9)$$
$$P(\text{even} \mid \geq 9)$$

13. **Currency** A dollar-bill changer in a snack machine was tested with 100 \$1-bills. Twenty-five of the bills were counterfeit. The results of the test are shown in the chart at the right.

Bill	Accepted	Rejected
Legal	69	6
Counterfeit	1	24

75
25

100

- a. What is the probability that a bill accepted by the changer is legal?
 b. What is the probability that a bill is rejected given that it is legal?
 c. What is the probability that a counterfeit bill is not rejected?

L/A
R/L
A/C

$$P_{L/A} = \frac{69}{70} = 99\% \quad (\text{accepted})$$

$$P_{R/L} = \frac{6}{75} = 8\%$$

$$P_{A/C} = \frac{1}{25} = 4\%$$

(AT)
~~AA~~
 (TH)
 (TT)

$$\frac{1}{3} \\
 \underline{2} \cdot \underline{2} \cdot \underline{2} = 8$$

G	G	G
G	G	B
G	B	G
B	G	G
G	B	B
B	G	B
B	B	G
B	B	B

} 8

21-42

2 2 2 2
16

$P_{3R/\#3R}$

RY

RY

RY

RY

2
8

~~RRRR~~
~~RRRy~~
RRyR
~~RYRR~~
~~yRRR~~

RRyy
RyyR
yyRR
yRRy
~~RYRy~~
yRyR

Ryyy
yRyy
~~yyRy~~
yyyR
yyyy

WB 13.5