Precalc 12.4

Determine whether a series is convergent or divergent
Use the comparison test

converge $r < 1$

diverge $r > 1$

ratio

ratio test $\underline{r = 1}$ $\underline{\shortparallel}$

general term

reference series

comparison test (n>1)

Total distance the ball bounces

video: fibanocci sequences
http://mathandmultimedia.com/2011/04/09/nature-by-numbers-video-by-cristobal-vila/

Ratio Test	Let a_n and a_{n+1} represent two consecutive terms of a series of positive terms. Suppose $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists and that $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. The series is convergent if $r < 1$ and divergent if $r > 1$. If $r = 1$, the test provides no information.

r<1 Convergent

r>1 Divergent

What if the ratio test is inconclusive? (r=1)

p. 789, also handout

Summary of Series for Reference

geom 1. **Convergent:** $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1} + \cdots, \ |r| < 1$

geom 2. **Divergent:** $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1} + \cdots, \ |r| > 1$

arith 3. **Divergent:** $a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \cdots$

harmonic 4. **Divergent:** $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n} + \cdots$ *This series is known as the harmonic series.*

p-series 5. **Convergent:** $1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots, \ p > 1$

Find general term

Compare to series that it most resembles

n>1 if conv, yours has to be less

if div, yours has to be more

Go straight to comparison test (probably bec. r=1...)

5 Use the comparison test to determine whether the following series are convergent or divergent.

a. $\frac{4}{5} + \frac{4}{7} + \frac{4}{9} + \frac{4}{11} + \cdots$

$$\begin{array}{ccccc} 1 & 2 & 3 & 4 \ldots & n \\ \frac{4}{5} & \frac{4}{7} & \frac{4}{9} & \frac{4}{11} & \frac{4}{2n+3} \end{array}$$

$n>1$

$\boxed{n=2}$

$$\frac{4}{2n+3} > \frac{1}{n}$$

div(?)

$$\cancel{\frac{4}{5} \quad \frac{1}{1}} \quad \frac{4}{7} > \frac{1}{2}$$

divergent

1 2 3 4

b. $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots$

$$\frac{n}{\ }$$

$$\frac{1}{(2n-1)^2}$$

p-series, conv.

$n>1$

$n=2$

$$\frac{1}{(2n-1)^2} < \frac{1}{n^2}$$

$$\frac{1}{3^2} < \frac{1}{2^2}$$

$$\frac{1}{9} < \frac{1}{4}$$