Precalc 12.4

Determine whether a series is convergent or divergent

Convergent (approaching a limit) some geometric series and some others

r < 1

Divergent (not approaching a limit) some geometric, all arithmetic and some others

r > 1

ratio

ratio test

12, 6, 3, 1.5 . . .

Quiz 12.1-12.2

$\frac{6}{12}$ $\frac{3}{6}$

general term

reference series

comparison test (n>1)

What kind of series is it?
What is the rule (r)?

+ ×

1 Determine whether each arithmetic or geometric series is convergent or divergent.

a. $-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\frac{1}{16}-\cdots$

$r = \frac{-1}{2}$

What kind of series is it?

b. $2 + 4 + 8 + 16 + \cdots$

$r = 2$ div.

if $r<1$ C

Pure geometric (what is r?)

Pure arithmetic d if $r>1$ D

Mixed...

Ratio Test	Let a_n and a_{n+1} represent two consecutive terms of a series of positive terms. Suppose $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ exists and that $r = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$. The series is convergent if $r < 1$ and divergent if $r > 1$. If $r = 1$, the test provides no information.

1. Find general term
a_n

2. Find the next (general) term after that
a_{n+1}

3. Apply the ratio test (limits)

4. Answer the question

$r<1$ C $\quad r=1$ no info

$r>1$ D

2 **Use the ratio test to determine whether each series is convergent or divergent.**

a. $\frac{1}{2} + \frac{2}{4} + \frac{3}{8} + \frac{4}{16} + \cdots$

Suggestion: make a table

1	2	3	4	...	n	$n+1$
$\frac{1}{2}$	$\frac{2}{4}$	$\frac{3}{8}$	$\frac{4}{16}$		$\boxed{\frac{n}{2^n}}$	$\boxed{\frac{n+1}{2^{n+1}}}$
2^1	2^2	2^3	2^4			

1. Find general term
a_n
2. Find the next (general) term
a_{n+1}
3. Apply the ratio test (limits) r=
4. Answer the question: Conv or div

$$\frac{n+1}{2^{n+1}} \cdot \frac{\cancel{2^n}}{n}$$

$$2^{n+1} = \cancel{2^n} \cdot 2^1$$

$$\frac{\frac{n+1}{n}}{2\frac{n}{n}} = \frac{1+0}{2 \cdot 1} = \frac{1}{2}$$

b. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots$

1	2	3	4	…		n	$n+1$
$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$			$\frac{n}{n+1}$	$\frac{n+1}{n+2}$

$$\frac{n+1}{n+2} \cdot \frac{n+1}{n} = \frac{\frac{n^2}{n^2} + \frac{2n}{n^2} + \frac{1}{n^2}}{\frac{n^2}{n^2} + \frac{2n}{n^2}} = \frac{1}{1} = 1$$

not enough info

S!

3 Use the ratio test to determine whether the series

$1 + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \cdots$ is convergent or divergent.

1	2	3	4	n	n+1
$\frac{1}{1}$	$\frac{1}{1\cdot 2}$	$\frac{1}{1\cdot 2\cdot 3}$	$\frac{1}{1\cdot 2\cdot 3\cdot 4}$	$\frac{1}{n!}$	$\frac{1}{(n+1)!}$

$$\frac{1}{(n+1)!}\cdot\frac{n!}{1} = \frac{\cancel{1}\cdot\cancel{1}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\ldots\cancel{n}}{\cancel{1}\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\ldots\cancel{n}(n+1)\cdot\cancel{1}} = \frac{\frac{1}{n}}{\frac{n}{n}+\frac{1}{n}}$$

$$= \frac{0}{1+0} = 0$$

Conv.

P789

Summary of Series for Reference

1. **Convergent:** $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1} + \cdots, |r| < 1$
2. **Divergent:** $a_1 + a_1r + a_1r^2 + \cdots + a_1r^{n-1} + \cdots, |r| > 1$
3. **Divergent:** $a_1 + (a_1 + d) + (a_1 + 2d) + (a_1 + 3d) + \cdots$
4. **Divergent:** $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots + \frac{1}{n} + \cdots$ *This series is known as the harmonic series.*
5. **Convergent:** $1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots, p > 1$ p-series

Will give you this

4 **Determine whether the series $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ is convergent or divergent.**

harmonic series: see p.789 trust me :)

5 **Use the comparison test to determine whether the following series are convergent or divergent.**

a. $\frac{4}{5}+\frac{4}{7}+\frac{4}{9}+\frac{4}{11}+\cdots$

Find the general term, compare to the series that it most resembles.
If n>1 (I always use n=2)
Convergent:
If each term< corresponding convergent series
Divergent:
If each term> corresponding divergent series

2n+3

b. $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$