Precalc 12.6

Use the binomial theorem to expand binomials
Find the nth term of a binomial expansion

binomial

coefficient

Pascal's triangle

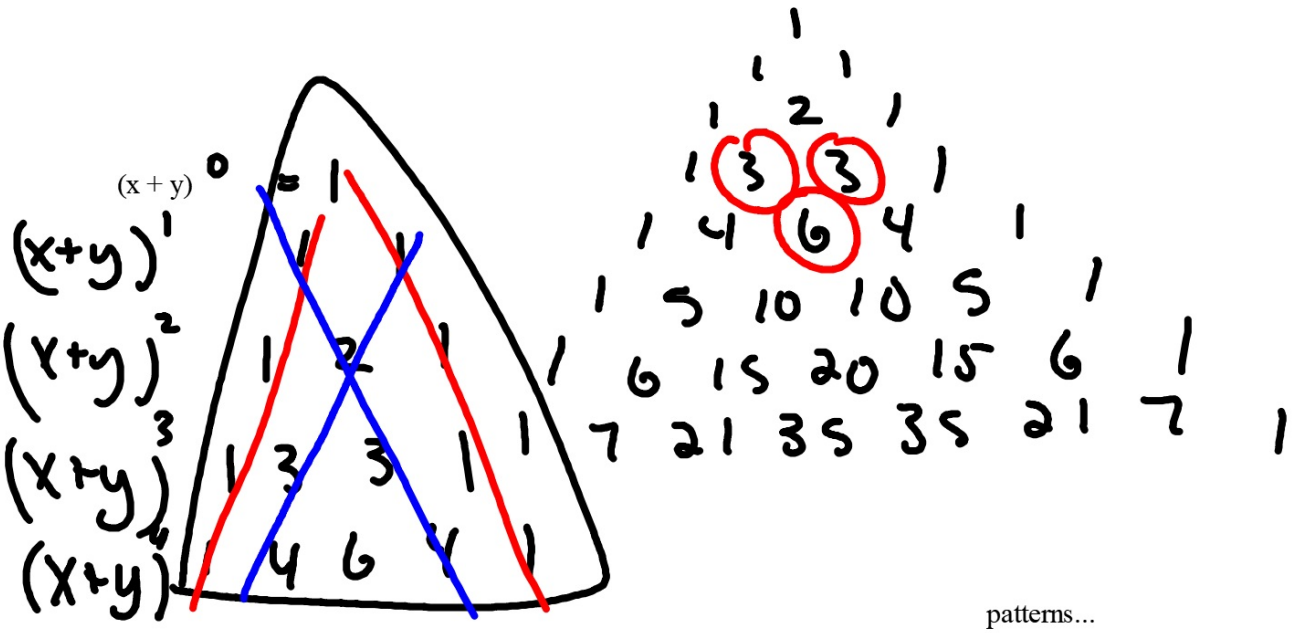
binomial expansion

expand vs. find a specific term

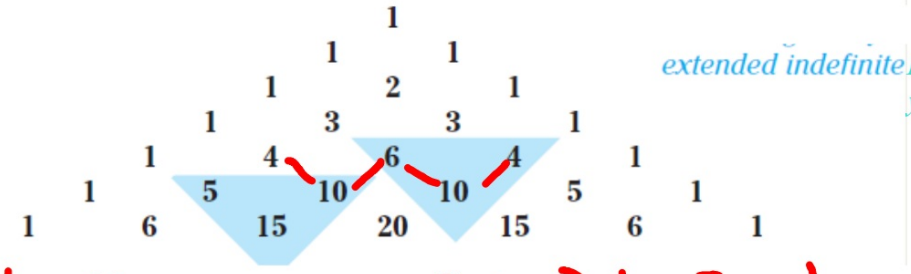
$$(x+3)^4$$

$$\begin{array}{r} x+3 \\ x+3 \\ \hline 3x+9 \\ x^2+3x \\ \hline x^2+6x+9 \\ x^2+6x+9 \\ \hline 9x^2\ 54x+81 \end{array}$$

$$\begin{array}{r} x^4\ 6x^3\ 36x^2\ 54x \\ x^4\ 6x^3\ 9x^2 \\ \hline x^4+12x^3+54x^2+108x+81 \end{array}$$


(x + y)^0 = 1
(x+y)^1
(x+y)^2
(x+y)^3
(x+y)^4
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
patterns...

Pascals triangle



extended indefinite

y

1 7 21 35 35 21 7 1

1 8 28 56 70 56 28 8 1

1 Use Pascal's triangle to expand each binomial.

a. $(x + y)^6$

$1x^6 + 6x^5y^1 + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + 1y^6$

$$(3p+-2q)^4$$

$$1(3p)^4 \quad 4(3p)^3(-2q)^1 \quad 6(3p)^2(-2q)^2 \quad 4(3p)^1(-2q)^3 \quad 1(-2q)^4$$

$$1 \cdot 81p^4 \quad 4 \cdot 27p^3 \cdot -2q \quad 6 \cdot 9p^2 \cdot 4q^2 \quad 4 \cdot 3 \cdot -8pq^3 \quad (-2)^4 q^4$$

$$81p^4 - 216p^3q + 216p^2q^2 - 96pq^3 + 16q^4$$

Use the Binomial Theorem to expand each binomial. (Pascal's triangle)

6. $(a+3)^7$ 7. $(5-y)^3$ 8. $(3p-2q)^4$

$(5+-1\cdot y)^3$

$1a^7 \quad 7a^6 3^1 \quad 21a^5 3^2 \quad 35a^4 3^3 \quad 35a^3 3^4 \quad 21a^2 3^5 \quad 7a 3^6 \quad 1\cdot 3^7$

$a^7 + 21a^6 + 189a^5 + 945a^4 + 2835a^3 + 5103a^2 + 5103a + 2187$

$1\cdot 5^3 \quad 3\cdot 5^2(-y)^1 \quad 3\cdot 5(-y)^2 \quad 1(-1y)^3$

$125 - 75y + 15y^2 - 1y^3$

4th term

3 Use the Binomial Theorem to expand $(2x - y)^6$.

$15 \cdot 4 \cdot 1 x^2 y^4$

$1(2x)^6 \quad 6(2x)^5(-1y)^1 \quad 15(2x)^4(-1y)^2 \quad 20(2x)^3(-1y)^3 \quad 15(2x)^2(-1y)^4 \quad 6(2x)(-1y)^5 \quad 1(-1y)^6$

$1 \cdot 64 x^6 \quad 6 \cdot 32 \cdot -1 x^5 y \quad 15 \cdot 16 \cdot 1 x^4 y^2 \quad 20 \cdot 8 \cdot -1 x^3 y^3 \quad 6 \cdot 2 \cdot -1 x y^5$

$64x^6 - 192x^5y + 240x^4y^2 - 160x^3y^3 + 60x^2y^2 - 12xy^5 + y^6$

b. $(3x + 2y)^7$

An equivalent form of the Binomial Theorem uses both sigma and factorial notation. It is written as follows, where n is a positive integer and r is a positive integer or zero.

$$(x + y)^n = \sum_{r=0}^{n} \frac{n!}{r!(n - r)!} x^{n-r} y^r$$

"Expand" vs find a specific term

"Expand" vs find a specific term

4 **Find the fifth term of $(4a + 3b)^7$.**

9. 6th term of $(a - b)^7$

10. 4th term of $(x + \sqrt{3})^9$