Precalc 12.5

Use sigma notation

sigma Σ

index of summation

typical element → general term

factorial

$$\sum_{n=1}^{17} (a_n)$$

$5! = {}^{*}\, 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = n(n-1)(n-2)(n-3)\ldots$

$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5$

Sigma Notation of a Series

For any sequence $a_1, a_2, a_3, \ldots$, the sum of the first k terms may be writter $\sum_{n=1}^{k} a_n$, which is read "the summation from $n = 1$ to k of a_n." Thus,

$$\sum_{n=1}^{k} a_n = a_1 + a_2 + a_3 + \cdots + a_k, \text{ where } k \text{ is an integer value.}$$

Have a plan...

1 **Write each expression in expanded form and then find the sum.**

a. $\sum_{n=1}^{4} (n^2 - 3)$ $= 18$

$n=1$	$n=2$	$n=3$	$n=4$
1^2-3	2^2-3	3^2-3	4^2-3
-2	1	6	13

3. Consider the series $\sum_{a}^{b} (-2j + 1)$. $10-2+1$

a. **Identify** the number of terms in this series.

b. **Write** a formula that determines the number of terms t in a finite series if the index of summation has a minimum value of a and a maximum value of b.

$b - a + 1$

c. Use the formula in part **b** to identify the number of terms in the series $\sum_{k=-2}^{3} \frac{1}{k+3}$.

$3 - -2 + 1$

d. Verify your answer in part c by writing the series $\sum_{k=-2}^{3} \frac{1}{k+3}$ in expanded forn

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-2	-1	0	1	2	3
$\frac{1}{-2+3}$	$\frac{1}{-1+3}$	$\frac{1}{0+3}$	$\frac{1}{1+3}$	$\frac{1}{2+3}$	$\frac{1}{3+3}$
1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$

+1

Write each expression in expanded form and then find the sum.

4. $\sum_{n=1}^{6} (n-3)$ $= 3$ 5. $\sum_{k=2}^{5} 4k$ 6. $\sum_{a=0}^{4} \frac{1}{2^a}$ 7. $\sum_{p=0}^{\infty} 5\left(\frac{3}{4}\right)^p$????

0	1	2	3	4	... n
$5\cdot\frac{3}{4}^0$	$5\cdot\frac{3}{4}^1$	$5\cdot\frac{3}{4}^2$	$5\cdot\frac{3}{4}^3$	$5\cdot\frac{3}{4}^4$	
$\rightarrow 5\cdot 1$	$5\cdot\frac{3}{4}$	$5\cdot\frac{3}{4}\cdot\frac{3}{4}$	$5\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$	$5\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}$	$5\cdot\frac{3}{4}^n$
$\rightarrow 5$	$\frac{15}{4}$	$\frac{45}{16}$	$\frac{135}{64}$	$\frac{405}{256}$	

notice that n does not always start at n=1...

$$\sum 5\cdot\frac{3}{4}^n = \frac{a_1}{1-r} = \frac{5}{1-\frac{3}{4}} = \frac{5}{\frac{1}{4}} = 5\cdot\frac{4}{1}$$

$a_1 = 5 \quad r = \frac{3}{4}$

$= 20$

b. $\sum_{n=1}^{\infty} 5\left(-\frac{2}{7}\right)^{n-1}$ *

Try writing the first several terms
Is it a series that you recognize?
(maybe a geometric series?...hint hint...)

$a_1 = 5$

$r = -\frac{2}{7}$

$\sum = \frac{5}{1 - -\frac{2}{7}} = \frac{5}{1+\frac{2}{7}} = \frac{5}{\frac{7}{7}+\frac{2}{7}} = \frac{5}{\frac{9}{7}} = 5 \cdot \frac{7}{9} = \frac{35}{9}$

1	2	3	4	... n
$5\left(-\frac{2}{7}\right)^0$	$5\left(-\frac{2}{7}\right)^1$	$5\left(-\frac{2}{7}\right)^2$	$5\left(-\frac{2}{7}\right)^3$	* $5\left(-\frac{2}{7}\right)^{n-1}$
5				

Express each series using sigma notation.

1 2 3 4 5

8. 5 + 10 + 15 + 20 + 25

$$\sum_{n=1}^{5} (5n)$$

1 2 3 4 ... n

9. 2 + 4 + 10 + 28

3^0+1 3^1+1 3^2+1 3^3+1

Remember: you can start with any integer for n

* $$\sum_{n=1}^{4} 3^{(n-1)}+1$$

* $$\sum_{n=0}^{3} 3^n+1$$

1 2 3 4

3 Express the series 15 + 24 + 35 + 48 + ⋯ + 143 using sigma notation.

16−1 25−1 36−1 49−1 144−1

What is the pattern?
you can start anywhere for n...

5	6	7	8	9
64−1	81−1	100−1	121−1	144−1

$$\sum_{n=1}^{9} (n+3)^2 - 1 \qquad \bigstar \sum_{n=4}^{12} n^2 - 1$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}$$

10. $2 - 4 - 10 - 16$

$$\begin{array}{ccc} -6 & -6 & -6 \end{array}$$

$-6n + 8$

$$\sum_{n=1}^{4} -6n + 8$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}$$

11. $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \frac{3}{32} + \cdots$ $\frac{3}{2^{n+1}}$

$$\begin{array}{cccc} 2^2 & 2^3 & 2^4 & 2^5 \end{array}$$

$$\sum_{n=1}^{\infty} \frac{3}{2^{n+1}}$$

$$\sum_{n=2}^{\infty} \frac{3}{2^n}$$

You can try graphing if it looks linear

***n* Factorial**	The expression $n!$ (n factorial) is defined as follows for n, an integer greater than zero. 5 $5(4)(3)(2)(1)$ $$n! = n(n-1)(n-2)\cdots 1$$

By definition $0! = 1$.

$0! = 1$

1 2 3 4 5

4 Express the series $\frac{2}{2} + \frac{4}{6} + \frac{6}{24} + \frac{8}{120} + \frac{10}{720}$ using sigma notation. $2n$

$2 \cdot 1 \quad 3 \cdot 2 \cdot 1 \quad 4 \cdot 3 \cdot 2 \cdot 1$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \quad 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$$\sum_{n=1}^{5} \frac{2n}{(n+1)!}$$

Consider numerator and denominator separately