Precalc15.3

Find values of integrals
Find the area under the curve of polynomial graphs

antiderivative F

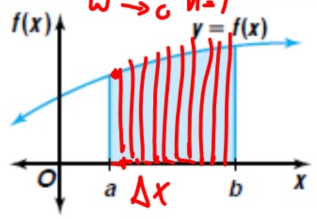
integral $\int$

definite integral $\int_0^3$

integration

area $= f(x) \cdot \Delta x$

$\lim_{w \to 0} \sum_{n=1}^{\infty} f(x) \cdot \Delta x$



p. 962 Inf. Series (Sum)

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{6n^5 + 15n^4 + 10n^3 - n}{30}$$

$$1^5 + 2^5 + 3^5 + \cdots + n^5 = \frac{2n^6 + 6n^5 + 5n^4 - n^2}{12}$$

How can we get a better estimate?

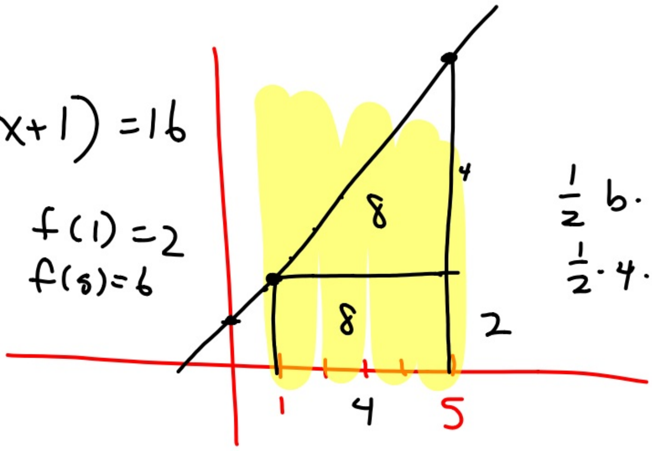
2. $\int_1^5 (x+1)\,dx$

Start with an easy one

$\int_1^5 (x+1) = 16$

$y = x + 1$

$f(1) = 2$
$f(5) = 6$



$\frac{1}{2} b \cdot h$

$\frac{1}{2} \cdot 4 \cdot 4$

vert bound.

width h

Definite Integral

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \text{ where } \Delta x = \frac{b-a}{n}.$$

top bound. (eq)

ht

$\frac{1}{2}$-ish

$\frac{1}{3}$-ish

p.963

1 Use limits to find the area of the region between the graph of $y = x^2$ and the x-axis from $x = 0$ to $x = 1$. That is, find $\boxed{\int_0^1 x^2\,dx}$. $y = x^2$

x_i = height of rect.

Δx = width of rect.

First find Δx. $\frac{1-0}{n} = \frac{1}{n}$ (b-a)/n (width of a bar)

(total width= starting point + additional)

a + iΔx

Then find x_i. $0 + \frac{1}{n}\cdot i = \frac{i}{n}$

$$\int_0^1 \left(\overset{ht}{x^2}\right)\left(\overset{w}{\Delta w}\right)$$

$(\quad)(\quad)$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^2\cdot\frac{1}{n} \qquad \frac{i}{n}\cdot\frac{i}{n}\cdot\frac{1}{n}$$

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^2}{n^3}$$

$$\lim_{n\to\infty}\frac{1^2}{n^3}+\frac{2^2}{n^3}+\frac{3^2}{n^3}\cdots$$

$$\lim_{n\to\infty}\frac{1}{\cancel{n^3}\;n^2}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)$$

$$\lim_{n\to\infty}\frac{2n^2+3n+1}{6n^2}$$

$$\lim_{n\to\infty}\frac{2n^2}{6n^2}+\frac{3n}{6n^2}+\frac{1}{6n^2}$$

$$\frac{1}{3}+\frac{1}{2n}+\frac{1}{6n} = \frac{1}{3}+0+0 = \frac{1}{3}$$

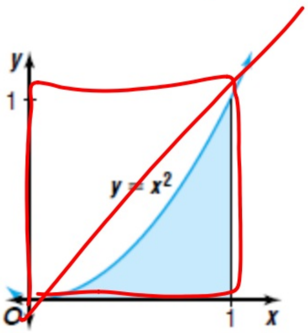
1 **Use limits to find the area of the region between the graph of $y = x^2$ and the x-axis from $x = 0$ to $x = 1$. That is, find $\int_0^1 x^2\,dx$.**

x^2 = height of rect.

Δx = width of rect.

First find Δx.

Then find x_i.



y * width

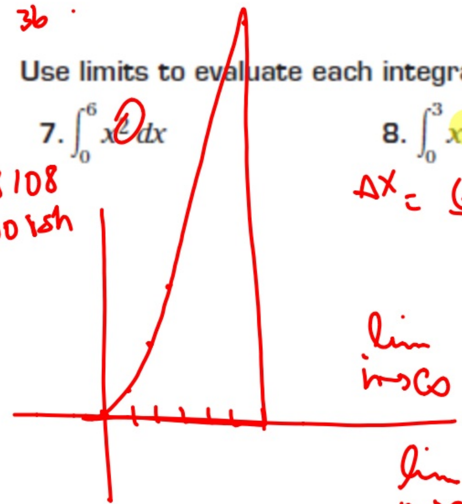
Definite Integral	$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$ where $\Delta x = \dfrac{b-a}{n}$.

36

Use limits to evaluate each integral.

7. $\int_0^6 x^2\,dx$ 8. $\int_0^3 x^3\,dx$

≈ 108
100 ish



$x_i = 0 + \frac{6 \cdot i}{n}$

$\Delta x = \frac{6-0}{n} = \frac{6}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{6i}{n}\right)^2 \left(\frac{6}{n}\right)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{216 i^2}{n^3}$$

$$\lim_{n\to\infty} \frac{216}{n^3}\left(1^2+2^2+3^2 \ldots\right)$$

$$\lim_{n\to\infty} \frac{216}{n^{\cancel{3}\,2}}\left(\frac{\cancel{n}(n+1)(2n+1)}{6}\right)$$

$$\lim_{n\to\infty} \frac{36}{n^2}\left(2n^2+3n+1\right)$$

$$\frac{72\cancel{n^2}}{\cancel{n^2}} + \frac{108\cancel{n}}{n^2} + \frac{36}{n^2}$$

$$\lim_{n\to\infty} 72 + \frac{108}{n} + \frac{36}{n^2}$$

$= 72$

2 Use limits to find the area of the region between the graph of $y = x^3$ and the x-axis from $x = 2$ to $x = 4$.

Strategy?