Precalc15.4 f' $\frac{dy}{dx}$

Use the fundamental theorem of calculus
Evaluate definite integrals of polynomial functions
Find indefinite integrals of polynomial functions

$f(x) = x^2 + \cancel{2}$

$\int$ Fundamental Theorem of Calculus (FTC)

$f'(x) = 2x$ $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

antiderivative

$F(x) = \frac{x^3}{3} + C$

integral

$\lim_{n \to \infty} \sum_{i=1}^{n} (\quad)(\quad)$

integration

definite

indefinite

Find the antiderivative of each function.

35. $f(x) = x^6$

$F(x) = \frac{x^7}{7}$

36. $f(x) = 3x + 4$

$F(x) = 3\left(\frac{x^2}{2}\right) + 4\left(\frac{x^1}{1}\right)$

$= \frac{3}{2}x^2 + 4x$

37. $f(x) = 4x^2 - 6x + 7$

$4\left(\frac{x^3}{3}\right) - 6\left(\frac{x^2}{2}\right) + 7x$

38. $f(x) = 12x^2 - 6x + 1$

antiderivative = indefinite integral

Example 2 Evaluate each indefinite integral.

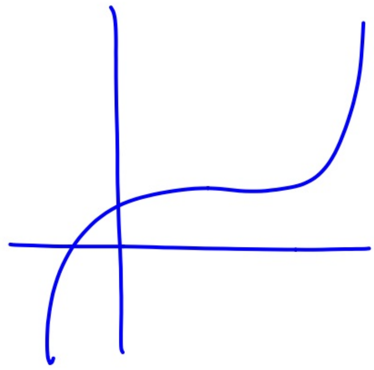
a. $\int 5x^2\,dx$

$f(x) = 5x^2$

$F(x) = 5\,\frac{x^3}{3}$

$= \frac{5}{3}x^3 + C$

b. $\int (4x^5 + 7x^2 - 4x)\, dx$



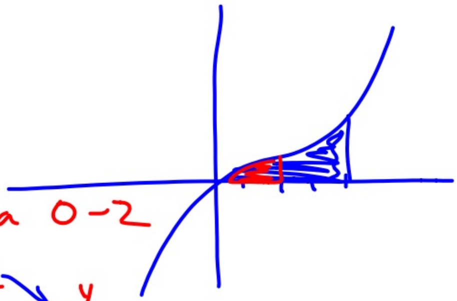
$\int 4x^5 + \int 7x^2 - \int 4x$

$4\left(\frac{x^6}{6}\right) + 7\left(\frac{x^3}{3}\right) - 4\left(\frac{x^2}{2}\right)$

$\frac{2}{3}x^6 + \frac{7}{3}x^3 - 2x^2 + C$

What did we do before?

Example 1 Evaluate $\int_2^4 x^3\,dx$.



+C?

Area 0-4 − Area 0-2

$$\int_0^4 \frac{x^4}{4} \quad - \int_0^2 \frac{x^4}{4}$$

$$\frac{4^4}{4}\left(\frac{256}{4}+c\right) - \frac{2^4}{4}\left(\frac{16}{4}+c\right)$$

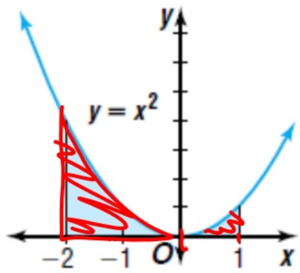
$$64 - 4 = 60$$

$$\int_2^4 x^3\,dx$$

$$\left.\frac{x^4}{4}\right|_2^4 \qquad 64 - 4 = 60$$

Definite integral (what happens to the +C?)

3 **Find the area of the shaded region.**



$$\int_{-2}^{0} x^2\,dx + \int_{0}^{1} x^2\,dx$$

$$\int_{-2}^{1} x^2\,dx = \frac{x^3}{3}\Big|_{-2}^{1} \quad \frac{1^3}{3} - \frac{(-2)^3}{3} = \frac{1}{3} - \frac{-8}{3} = \frac{9}{3} = 3$$

Fundamental Theorem of Calculus	If $F(x)$ is the antiderivative of the continuous function $f(x)$, then $$\int_a^b f(x)\,dx = F(b) - F(a).$$

eg

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

Evaluate each definite integral.

10. $\int_1^3 2x^3\,dx$

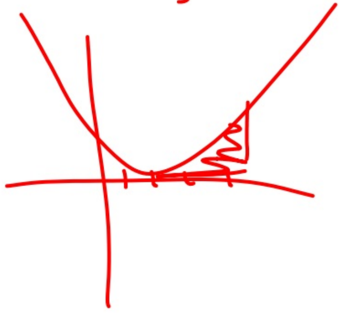
$$2\left(\frac{x^4}{4}\right) = \frac{x^4}{2}\Bigg|_1^3 = \frac{3^4}{2} - \frac{1^4}{2}$$

$$= 40{,}5 = \frac{1}{2}$$

$$= 40$$

11. $\int_1^4 (x^2 - x + 6)\, dx$

$$\frac{x^3}{3} - \frac{x^2}{2} + \frac{6x}{1} \Bigg|_1^4 \quad \left(\frac{4^3}{3} - \frac{4^2}{2} + \frac{6 \cdot 4}{1}\right) - \left(\frac{1^3}{3} - \frac{1^2}{2} + 6\right)$$



$$\left(\frac{64}{3} - \frac{16}{2} + 24\right) - \left(\frac{1}{3} - \frac{1}{2} + 6\right)$$

$$37\tfrac{1}{3} - 5\tfrac{5}{6}$$

$$31\tfrac{1}{2}$$

1. **Explain** the difference between $\int f(x)\,dx$ and $\int_a^b f(x)\,dx$. ( ) - ( )

+c

x^3 $\frac{x^4}{4}$

	Power Rule:	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$, where n is a rational number and $n \neq -1$.
Antiderivative Rules	Constant Multiple of a Power Rule:	$\int kx^n\,dx = k \cdot \frac{1}{n+1}x^{n+1} + C$, where k is a constant, n is a rational number, and $n \neq -1$.
	Sum and Difference Rule:	$\int (f(x) \pm g(x))\,dx = \underline{\int f(x)\,dx} \pm \int \underline{g(x)\,dx}$

$4(x^2)$

$4\left(\frac{x^3}{3}\right)$

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