Geometry 2.3
Analyze statements in if-then form
Write the converse, inverse, and contrapositive of conditional statements

$p \to q$
conditional statement — biconditional
hypothesis if — if and only if
conclusion then — "iff"
related conditional
converse $q \to p$
inverse $\sim p \to \sim q$
contrapositive $\sim q \to \sim p$
logically equivalent
Same truth value

KeyConcept Conditional Statement

Words	Symbols
An if-then statement is of the form *if p, then q.*	$p \rightarrow q$ read *if p then q*, or *p implies q*
The hypothesis of a conditional statement is the phrase immediately following the word *if.*	p
The conclusion of a conditional statement is the phrase immediately following the word *then.*	q

2 Related Conditionals

There are other statements that are based on a given conditional statement. These are known as **related conditionals**.

KeyConcept Related Conditionals

Words	Symbols	Examples
A conditional statement is a statement that can be written in the form *if p, then q*.	$p \to q$	If $m\angle A$ is 35, then $\angle A$ is an acute angle.
The **converse** is formed by exchanging the hypothesis and conclusion of the conditional.	$q \to p$	If $\angle A$ is an acute angle, then $m\angle A$ is 35.
The **inverse** is formed by negating both the hypothesis and conclusion of the conditional.	$\sim p \to \sim q$	If $m\angle A$ is *not* 35, then $\angle A$ is *not* an acute angle.
The **contrapositive** is formed by negating both the hypothesis and the conclusion of the converse of the conditional.	$\sim q \to \sim p$	If $\angle A$ is *not* an acute angle, then $m\angle A$ is *not* 35.

If you live in Sioux Falls, then you live in SD.

A conditional and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional are either both true or both false. Statements with the same truth values are said to be **logically equivalent**.

KeyConcept Logically Equivalent Statements

- A conditional and its contrapositve are logically equivalent.
- The converse and inverse of a conditional are logically equivalent.

$\sim p \rightarrow \sim q$

$\sim q \rightarrow \sim p$

Example 2 **Write each statement in if-then form.**

5. Sixteen-year-olds are eligible to drive.
6. Cheese contains calcium.
7. The measure of an acute angle is between 0 and 90.
8. Equilateral triangles are equiangular.

if cheese → Ca

~~if Ca then cheese~~

Con: if 16 then drive. T

Conv if drive then 16 (F)

Inv. if not 16 not eligible to drive (F)

CP if not drive then not 16 T

Example 3 **Determine the truth value of each conditional statement. If *true*, explain your reasoning. If *false*, give a counterexample.**

10. If $x^2 = 16$, then $x = 4$. F if $x = -4$
11. If you live in Charlotte, then you live in North Carolina. T
12. If tomorrow is Friday, then today is Thursday. T
13. If an animal is spotted, then it is a Dalmatian. F could be cheetah
14. If the measure of a right angle is 95, then bees are lizards. F T
15. If pigs can fly, then $2 + 5 = 7$. F T

Remember: benefit of the doubt…

Whiteboards

0, 1, 2, 3 ... ...-3, -2, -1 0, 1, 2, 3 . .

if whole then integer T

conv if int then whole F

inv. if not whole then not int. F

contr. if not int then not whole T

Example 4 CONSTRUCT ARGUMENTS Write the converse, inverse, and contrapositive of each true conditional statement. Determine whether each related conditional is *true* or *false*. If a statement is false, find a counterexample.

F p q F

16. If a number is divisible by 2, then it is divisible by 4.

17. All whole numbers are integers

$\frac{1}{300}$ $\frac{10}{300}$ $\frac{1}{30}$

if not ÷ by 4 then not ÷ by 2

Start by writing in if/then form.

E → 60

If a triangle is equilateral, then each angle is 60 degrees.

If a triangle has three 60 degree angles, then it is equilateral.

60 → E

"If and only if...iff"

$$\left.\begin{array}{l} p \to q \\ q \to p \end{array}\right\} T$$

KeyConcept Biconditional Statement

Words	A biconditional statement is the conjunction of a conditional and its converse.
Symbols	$(p \rightarrow q) \wedge (q \rightarrow p) \rightarrow (p \leftrightarrow q)$, read *p if and only if q*

If and only if can be abbreviated *iff*.

Examples

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

a. **An angle is a right angle if and only if its measure is 90.**

if 90 → rt ∠

Conditional: If an angle measures 90, then the angle is right.

if rt∠ → 90

Converse: If an angle is right, then the angle measures 90.

Both the conditional and the converse are true, so the biconditional is true.

b. **$x > -2$ iff x is positive.**

if x pos. → >-2 T

Conditional: If x is positive, then $x > -2$.

if x>-2 then pos. F

Converse: If $x > -2$, then x is positive.

Let $x = -1$. Then $-1 > -2$, but -1 is not positive. So, the biconditional is false.

Write both statement & converse. Are they both true?

Exercises

Write each biconditional as a conditional and its converse. Then determine whether the biconditional is *true* or *false*. If false, give a counterexample.

1. Two angles are complements if and only if their measures have a sum of 90.
2. There is no school if and only if it is Saturday.

if 90 → comp
if comp → 90

if S → no school
if no school → S