Geometry   4.3 $\triangle ABC \cong \triangle SQR$

Name and use corresponding parts of congruent polygons
Prove triangles congruent using the definition of congruence
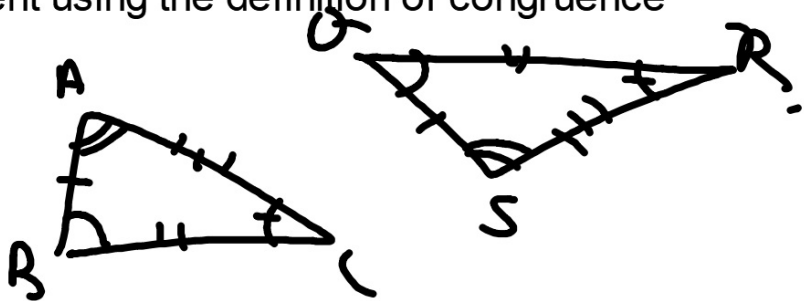congruent
corresponding parts
reflexive property
symmetric property
transitive property

activity: triangle cutouts
            exploragons

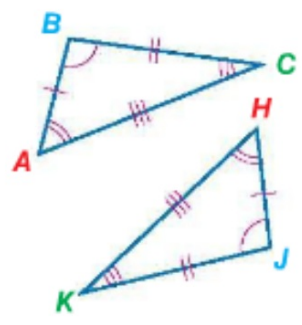## KeyConcept Definition of Congurent Polygons

Words	Two polygons are congruent if and only if their corresponding parts are congruent.

Model

Example

Corresponding Angles

$\angle A \cong \angle H$ $\angle B \cong \angle J$ $\angle C \cong \angle K$
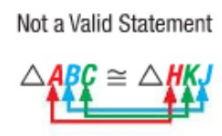
Corresponding Sides

$\overline{AB} \cong \overline{HJ}$ $\overline{BC} \cong \overline{JK}$ $\overline{AC} \cong \overline{HK}$

Congruence Statement

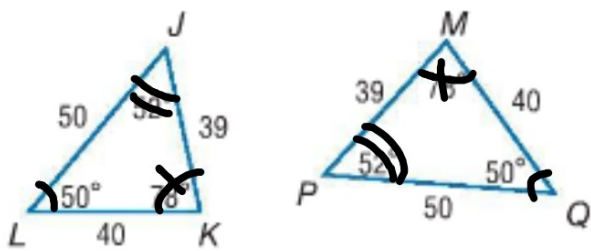$\triangle ABC \cong \triangle HJK$

CBA KJH

Valid Statement

$\triangle BCA \cong \triangle JKH$

Not a Valid Statement

$\triangle ABC \cong \triangle HKJ$

1B.

Triangle JKL: J at top, 50 on side JL, 39 on side JK, 50° angle at L, 40 on side LK.

Triangle PMQ: M at top, 39 on side PM, 40 on side MQ, 52 at P, 50° at Q, 50 on side PQ.

$\triangle JKL \cong \triangle PMQ$

iff

The phrase "if and only if" in the congruent polygon definition means that both the conditional and its converse are true. So, if two polygons are congruent, then their corresponding parts are congruent. For triangles, we say *Corresponding parts of congruent triangles are congruent*, or CPCTC.
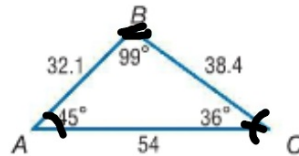
$$26 + x = 38.4$$
$$-26 \qquad -26$$
$$x = 12.4$$

Identify corresponding parts
Write/solve an equation

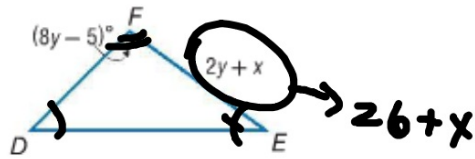## Example 2 Use Corresponding Parts of Congruent Triangles

In the diagram, $\triangle ABC \cong \triangle DFE$. Find the values of x and y.

$$8y - 5 = 99$$
$$+5 \quad +5$$
$$8y = 104 \qquad y = 13$$
$$\frac{8y}{8} = \frac{104}{8}$$

$$x = 12.4$$
$$y = 13$$

Triangle ABC: B at top (99°), A (45°), C (36°); sides 32.1 (AB), 38.4 (BC), 54 (AC).

Triangle DFE: F at top ((8y − 5)°, 2y + x), D and E at base; 26 + x.

2. In the diagram, $\triangle RSV \cong \triangle TVS$. Find the values of x and y.

R — 2y − 1 — S
12
78° V 24.5 24 T
x°
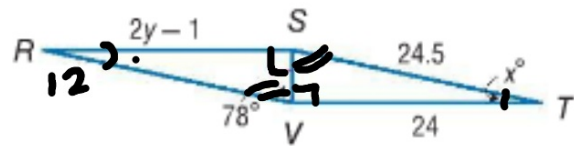
$$2y - 1 = 24$$
$$\quad +1 \quad +1$$
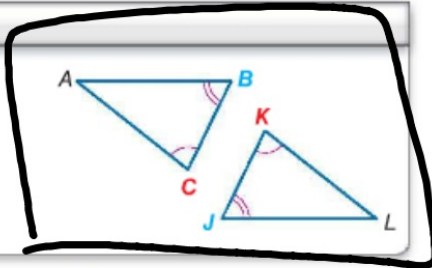$$2y = 25$$

$$x = 12$$
$$y = 12.5$$

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2 **Prove Triangles Congruent** The Triangle Angle-Sum Theorem you learned in Lesson 4-2 leads to another theorem about the angles in two triangles.

Theorem 4.3 **Third Angles Theorem**

Words: If two angles of one triangle are congruent to two angles of a second triangle, then the third angles of the triangles are congruent.

Example: If $\angle C \cong \angle K$ and $\angle B \cong \angle J$, then $\angle A \cong \angle L$.

You will prove this theorem in Exercise 21.

given
∴
prove

Given $\angle C \cong \angle K$, $\angle B \cong \angle J$
Prove: $\angle A \cong \angle L$

1. $\angle C \cong \angle K$; $\angle B \cong \angle J$	1. given
2. $\angle A + \angle B + \angle C = 180$ $\angle J + \angle K + \angle L = 180$	2. △ sum th.
3. $\angle A + \angle B + \angle C = \angle J + \angle K + \angle L$ $-\angle B \ -\angle C \quad \angle J \ -\angle K$	3. subs.
4. $\angle A \cong \angle L$	4. Subtract.

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Prove that all corresp parts are congruent

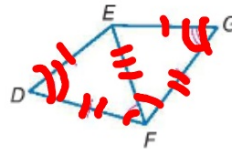p. 257

Example 4 Prove That Two Triangles are Congruent

Write a two-column proof.

Given: $\overline{DE} \cong \overline{GE}, \overline{DF} \cong \overline{GF}, \angle D \cong \angle G,$
$\angle DFE \cong \angle GFE$

Prove: $\triangle DEF \cong \triangle GEF$

Proof:

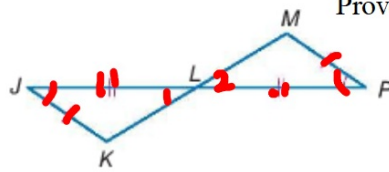| Statements	Reasons
1. $\overline{DE} \cong \overline{GE}, \overline{DF} \cong \overline{GF}$	1. Given
2. $\overline{EF} \cong \overline{EF}$	2. Reflexive Property of Congruence
3. $\angle D \cong \angle G, \angle DFE \cong \angle GFE$	3. Given
4. $\angle DEF \cong \angle GEF$	4. Third Angles Theorem
5. $\triangle DEF \cong \triangle GEF$	5. Definition of Congruent Polygons

4. Write a two column proof.

Given: $\angle J \cong \angle P$, $\overline{JK} \cong \overline{PM}$,
$\overline{JL} \cong \overline{PL}$, and L bisects $\overline{KM}$.

Prove: $\triangle JLK \cong \triangle PLM$

Prove all pairs of corresp. parts congruent

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1. $\angle J \cong \angle P$ $\overline{JK} \cong \overline{PM}$ $\overline{JL} \cong \overline{PL}$, L bisects	1. given
2. $\angle 1 \cong \angle 2$	2. VA
3. $\angle K \cong \angle M$	3. 3rd $\angle$ theor.
4. $\overline{KL} \cong \overline{ML}$	4. def bisect.
5. $\triangle JLK \cong \triangle PLM$	5. all corresp. parts $\cong$

Like congruence of segments and angles, congruence of triangles is reflexive, symmetric, and transitive.

Theorem 4.4 **Properties of Triangle Congruence**

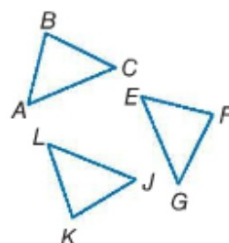**Reflexive Property of Triangle Congruence**

$\triangle ABC \cong \triangle ABC$

Symmetric Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$, then $\triangle EFG \cong \triangle ABC$.

Transitive Property of Triangle Congruence

If $\triangle ABC \cong \triangle EFG$ and $\triangle EFG \cong \triangle JKL$, then $\triangle ABC \cong \triangle JKL$.

WB 4.3
Skills + prac.