Geometry 10.3

Recognize and use relationships between arcs and chords

Recognize and use diameter relationships

⊙A

arc

(major arc) $\overset{\frown}{XCY}$

(minor arc) $\overset{\frown}{XY}$

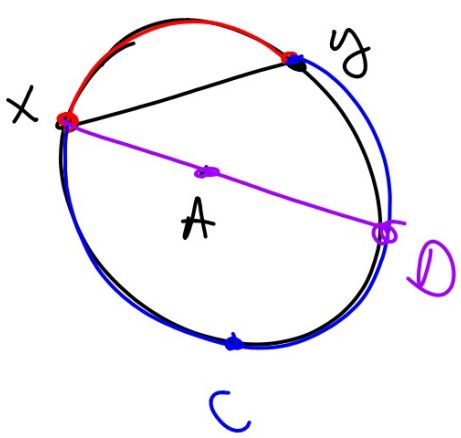
chord $\overline{XY}$

diameter $\overline{XD}$

radius

equidistant

same distance

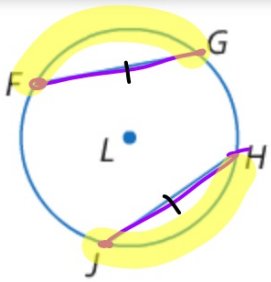


if
f

## Theorem 10.2

**Words** In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.

**Example** $\overset{\frown}{FG} \cong \overset{\frown}{HJ}$ if and only if $\overline{FG} \cong \overline{HJ}$.




$\overline{FG} \cong \overline{JH}$

$\overset{\frown}{FG} \cong \overset{\frown}{JH}$

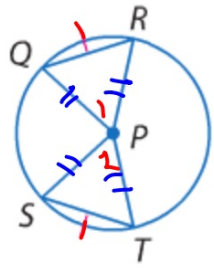
If arcs are = then chords are =
Prove it using SAS

**Proof** Theorem 10.2 (part 1)

**Given:** $\odot P$; $\overset{\frown}{QR} \cong \overset{\frown}{ST}$

**Prove:** $\overline{QR} \cong \overline{ST}$

**Proof:**




| Statements | Reasons |
| --- | --- |
| 1. $\overset{\frown}{QR} \cong \overset{\frown}{ST}$ | 1. given |
| 2. $\overline{PQ} \cong \overline{PR} \cong \overline{PS} \cong \overline{PT}$ | 2. radii of $\odot P$ |
| 3. $\angle 1 \cong \angle 2$ | 3. central $\angle$s w same arc |
| 4. $\triangle QRP \cong \triangle TSP$ | 4. SAS |
| 5. $\overline{QR} \cong \overline{ST}$ | 5. CPCTC |

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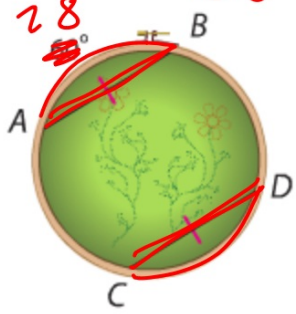
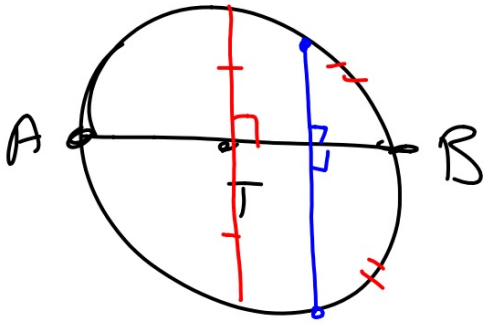
## Real-World Example 1 Use Congruent Chords to Find Arc Measure

**CRAFTS** In the embroidery hoop, $\overline{AB} \cong \overline{CD}$ and $m\overset{\frown}{AB} = 60$. Find $m\overset{\frown}{CD}$.



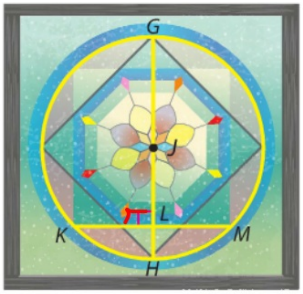

**1.** If $m\overset{\frown}{AB} = 78$ in the embroidery hoop, find $m\overset{\frown}{CD}$.

A diameter perpendicular to a chord bisects the chord (and its arc)



## Real-World Example 4 Use a Diameter Perpendicular

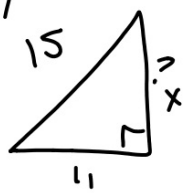
**STAINED GLASS** In the stained glass window, diameter $\overline{GH}$ is 30 inches long and chord $\overline{KM}$ is 22 inches long. Find $JL$.

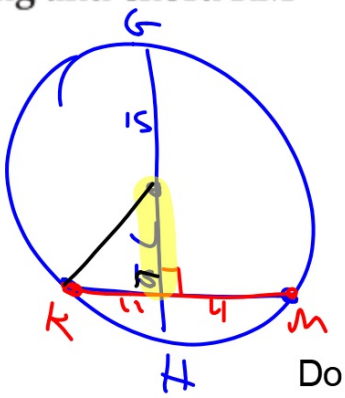



$11^2 + x^2 = 15^2$

$x^2 = 104$

$x \approx 10.2$



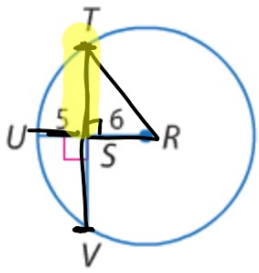




Do we know the radius of the circle?

**GuidedPractice**

18.44

**4.** In $\odot R$, find $TV$. Round to the nearest hundredth.



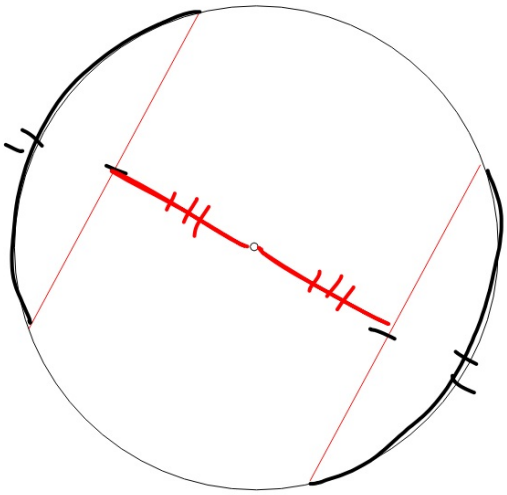

9.22 x 11

$x^2 + 6^2 = 11^2$

6

TV = ?

Do we know the radius?



Are the segments the same length?

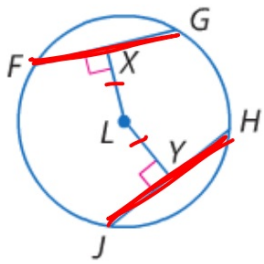
equidistant from center

iff

## Theorem 10.5

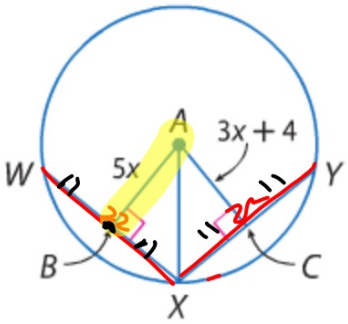
**Words** In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.

**Example** $\overline{FG} \cong \overline{JH}$ if and only if $LX = LY$.



## Example 5 Chords Equidistant from Center

**ALGEBRA** In $\odot A$, $WX = XY = 22$. Find $AB$. $= 10$




$5x = 3x + 4$

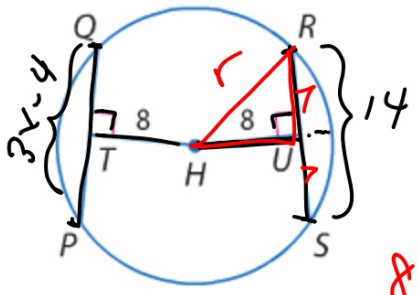
$-3x \quad -3x$

$2x = 4$

$x = 2$

## GuidedPractice

**5.** In $\odot H$, $PQ = 3x - 4$ and $RS = 14$. Find $x$.




$3x - 4 = 14$

$+4 \quad +4$

$3x = 18$

$x = 6$

$8^2 + 7^2 = r^2$