Geometry 6.5

Recognize and apply the properties of rhombi and squares

Determine whether a given quadrilateral is a rectangle, rhombus or square

rhombus *(diamond is not a geometry term!)*

square

diagonal

perpendicular

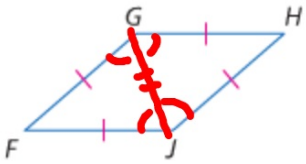
converse

little book--rhombus, square

activ: exploragons

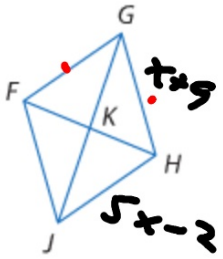
Exploragons
parallelogram
rhombus

1 Properties of Rhombi and Squares A **rhombus** is a parallelogram with all four sides congruent. A rhombus has all the properties of a parallelogram and the two additional characteristics described in the theorems below.



1. Diagonals of a rhombus are perpendicular.
2. Diagonals of a rhombus form congruent triangles.
3. Diagonals of a rhombus bisect opposite angles.

Example 1 Use Properties of a Rhombus



The diagonals of rhombus $FGHJ$ intersect at K. Use the given information to find each measure or value.

a. If $m\angle FJH = 82$, find $m\angle KHJ$. $= 49°$

$5x - 2 = x + 9$

$-x \quad +2 \quad -x \quad +2$

$4x = 11$

$x = \frac{11}{4}$

b. **ALGEBRA** If $GH = x + 9$ and $JH = 5x - 2$, find x.

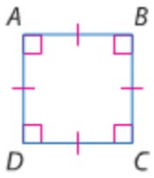
GuidedPractice

Refer to rhombus $FGHJ$ above.

1A. If $FK = 5$ and $FG = 13$, find KJ.

1B. ALGEBRA If $m\angle JFK = 6y + 7$ and $m\angle KFG = 9y - 5$, find y.

A **square** is a parallelogram with four congruent sides and four right angles. Recall that a parallelogram with four right angles is a rectangle, and a parallelogram with four congruent sides is a rhombus. Therefore, a parallelogram that is both a rectangle and a rhombus is also a square.



Square *ABCD*

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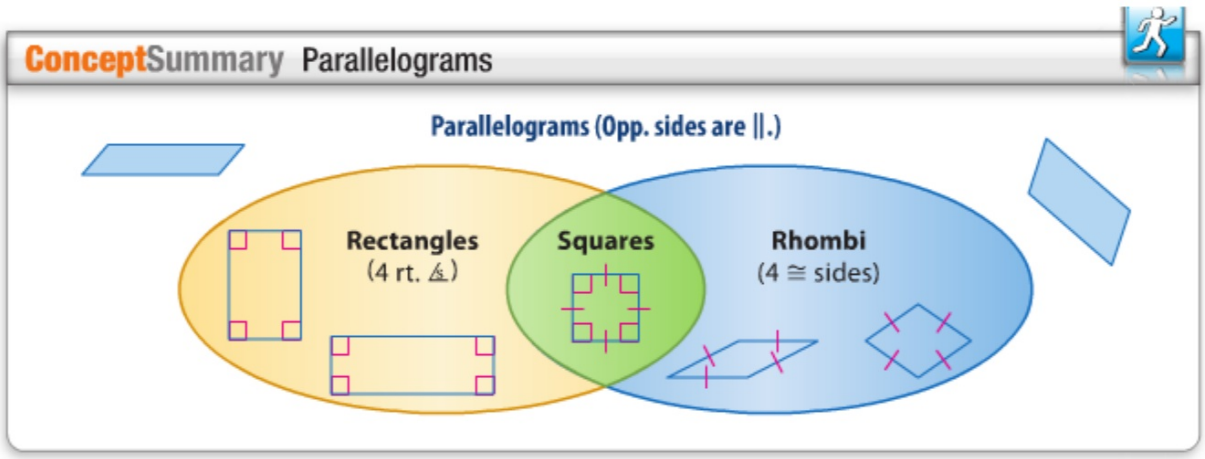
ConceptSummary Parallelograms

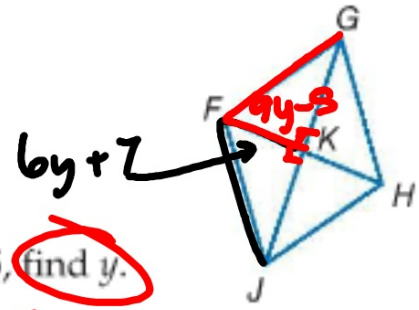


GuidedPractice

Refer to rhombus *FGHJ* above.

1A. If $FK = 5$ and $FG = 13$, find KJ.

1B. ALGEBRA If $m\angle JFK = 6y + 7$ and $m\angle KFG = 9y - 5$, find y.



$9y - 5 = 6y + 7$

$-6y + 5 \quad -6y + 5$

$\frac{3y}{3} = \frac{12}{3}$

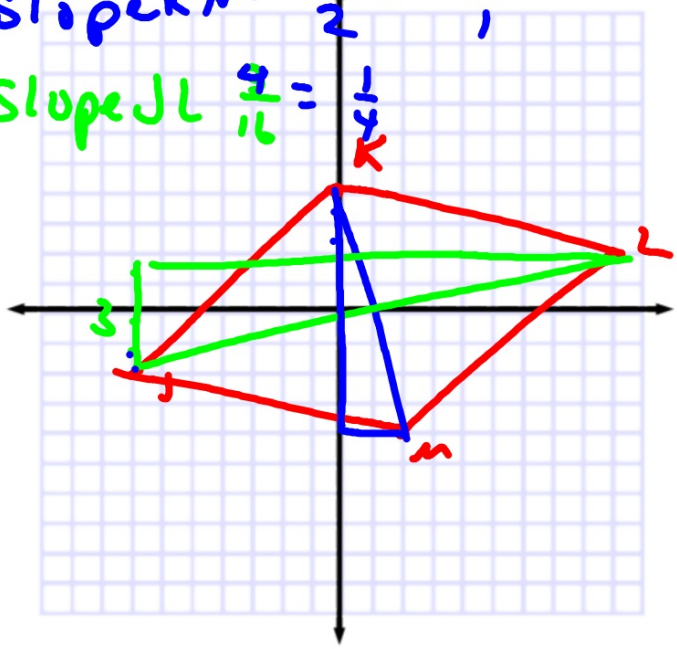
$y = 4$

Example 4 Classify Quadrilaterals Using Coordinate Geometry

COORDINATE GEOMETRY Determine whether $\square JKLM$ with vertices $J(-7, -2)$, $K(0, 4)$, $L(9, 2)$, and $M(2, -4)$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.

Slope KM $\frac{-6}{2} = \frac{-4}{1}$

Slope JL $\frac{2}{16} = \frac{1}{4}$

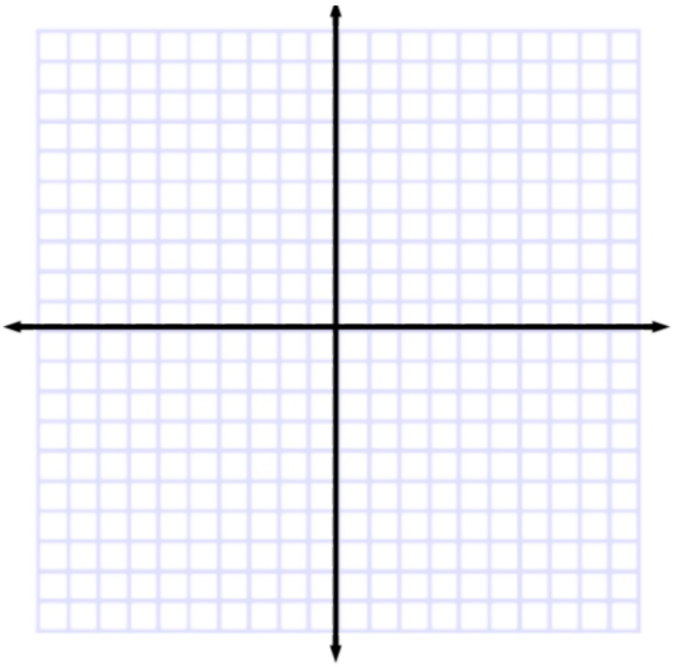


rhombus
no rect
no square

Graph and make a preliminary decision.
Do the math and prove it.
(Can't go by eyeball)

GuidedPractice

4. Given $J(5, 0)$, $K(8, -11)$, $L(-3, -14)$, $M(-6, -3)$, determine whether parallelogram $JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.



StudyTip

Common Misconception

Theorems 6.17, 6.18, and 6.19 apply only if you already know that a quadrilateral is a parallelogram.

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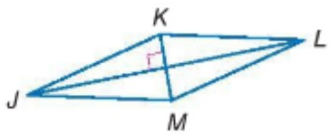
Theorems Conditions for Rhombi and Squares

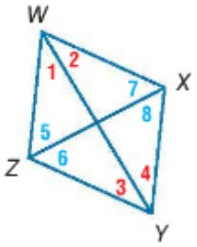
6.17 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (Converse of Theorem. 6.15)

Example If $\overline{JL} \perp \overline{KM}$, then $\square JKLM$ is a rhombus.



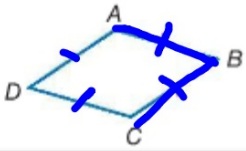
6.18 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (Converse of Theorem. 6.16)

Example If $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$, or $\angle 5 \cong \angle 6$ and $\angle 7 \cong \angle 8$, then $\square WXYZ$ is a rhombus.



6.19 If a pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

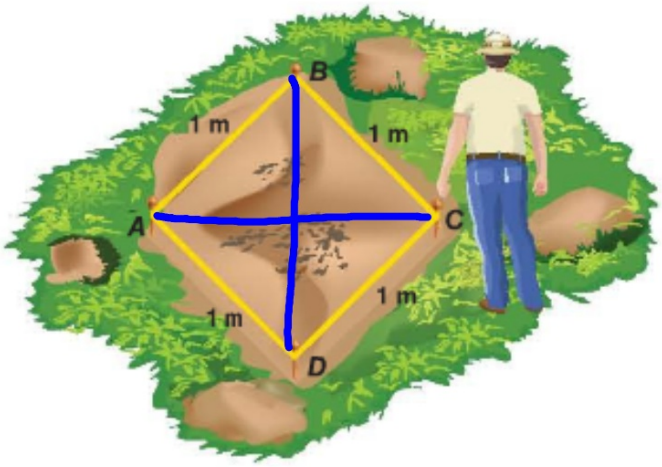
Example If $\overline{AB} \cong \overline{BC}$, then $\square ABCD$ is a rhombus.



6.20 If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Real-World Example 3 Use Conditions for Rhombi and Squares

ARCHAEOLOGY The key to the successful excavation of an archaeological site is accurate mapping. How can archaeologists be sure that the region they have marked off is a 1-meter by 1-meter square?



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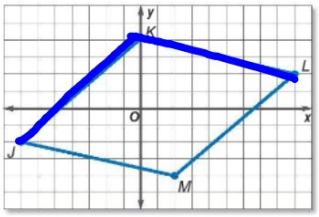


StudyTip

Square and Rhombus

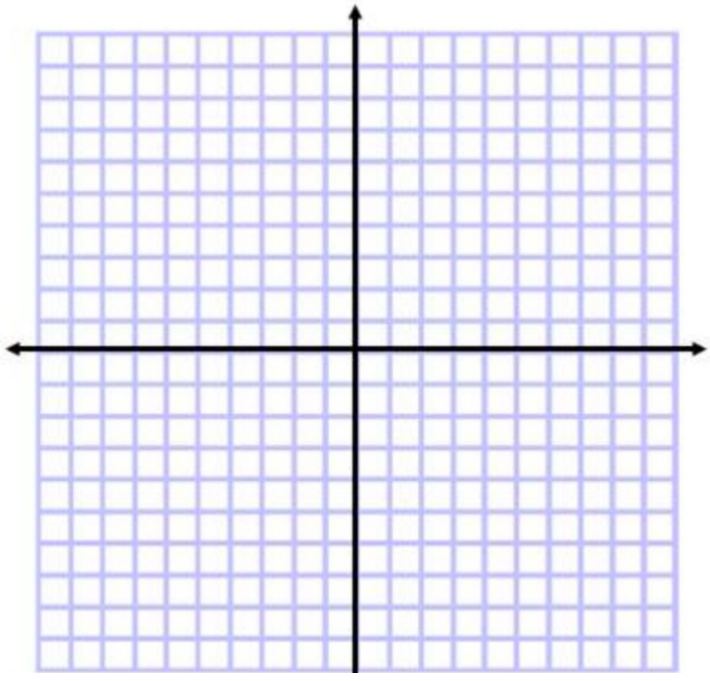
A square is a rhombus, but a rhombus is not necessarily a square.

Every square is a rhombus.

Not every rhombus is a square.

GuidedPractice

4. Given $J(5, 0)$, $K(8, -11)$, $L(-3, -14)$, $M(-6, -3)$, determine whether parallelogram $JKLM$ is a *rhombus*, a *rectangle*, or a *square*. List all that apply. Explain.



6.5 p. 435

7-33 odd