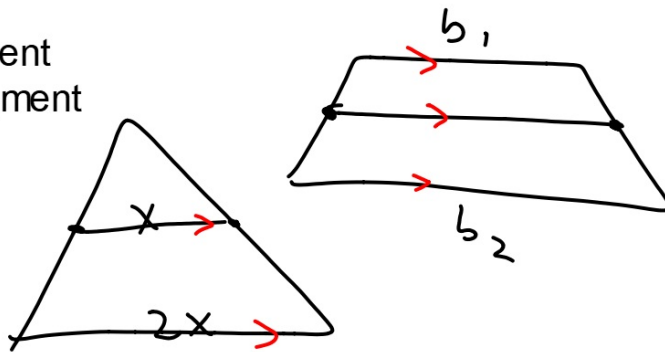Geometry 7.4
Use proportional parts within triangles
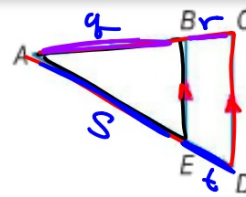Use proportional parts with parallel lines

triangle midsegment
trapezoid midsegment
parallel
transversal
midsegment
proportion

b_1

x

$2x$

b_2

Theorem 7.5 Triangle Proportionality Theorem

If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.

Example If $\overline{BE} \parallel \overline{CD}$, then $\dfrac{AB}{BC} = \dfrac{AE}{ED}$.

$$\frac{q}{r} = \frac{s}{t}$$

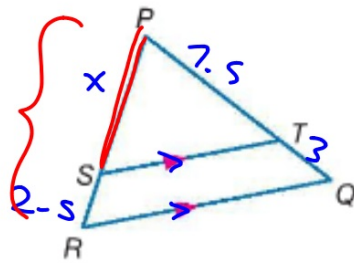**Example 1** Find the Length of a Side

In $\triangle PQR$, $\overline{ST} \parallel \overline{RQ}$. If $PT = 7.5$, $TQ = 3$, and $SR = 2.5$, find PS.

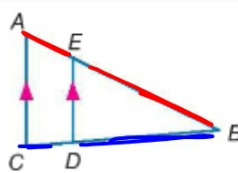$$\frac{x}{2.5} = \frac{7.5}{3}$$

$$3x = 18.75$$

$$x = 6.25$$

$$PR = 8.75$$

Theorem 7.6 Converse of Triangle Proportionality Theorem

If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.

Example If $\dfrac{AE}{EB} = \dfrac{CD}{DB}$, then $\overline{AC} \parallel \overline{ED}$.
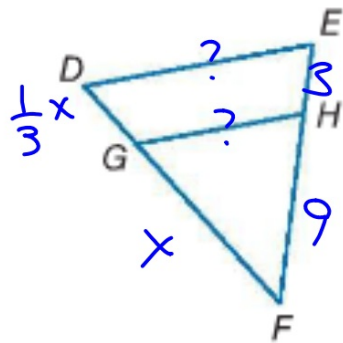
Example 2 Determine if Lines are Parallel

In $\triangle DEF$, $EH = 3$, $HF = 9$, and DG is one-third the length of $\overline{GF}$. Is $\overline{DE} \parallel \overline{GH}$?   yes
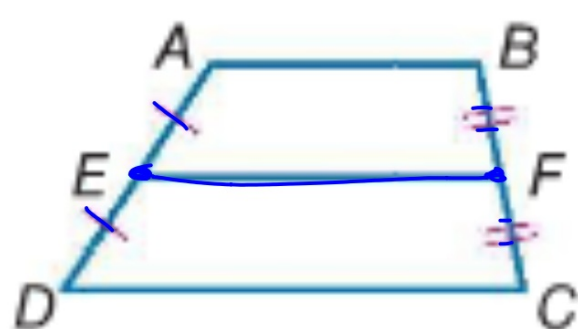
$$\frac{\frac{1}{3}x}{x} \overset{?}{=} \frac{3}{9}$$
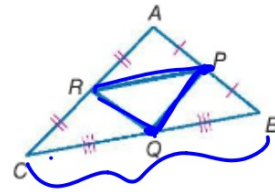
$$3x = \frac{1}{3}x \cdot 9$$

$$3x = 3x$$

$$\overline{EF} \parallel \overline{AB} \parallel \overline{DC}$$

$$EF = \frac{1}{2}(AB + DC)$$

A **midsegment of a triangle** is a segment with
endpoints that are the midpoints of two sides of
the triangle. Every triangle has three midsegments.
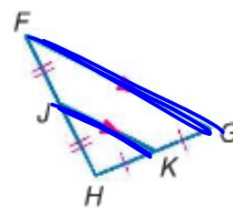The midsegments of $\triangle ABC$ are $\overline{RP}, \overline{PQ}, \overline{RQ}$.

A special case of the Triangle Proportionality
Theorem is the Triangle Midsegment Theorem.

Theorem 7.7 **Triangle Midsegment Theorem**

A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.

Example If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.
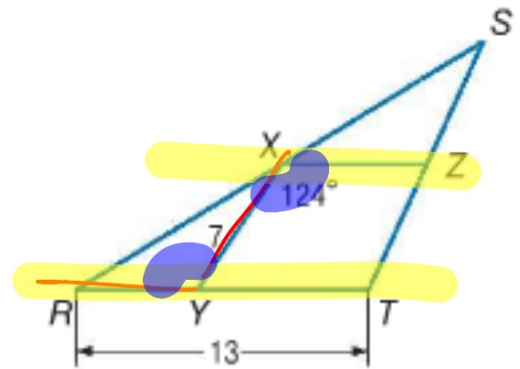
You will prove Theorem 7.7 in Exercise 32.

Example 3 Use the Triangle Midsegment Theorem

In the figure, $\overline{XY}$ and $\overline{XZ}$ are midsegments of $\triangle RST$. Find each measure.
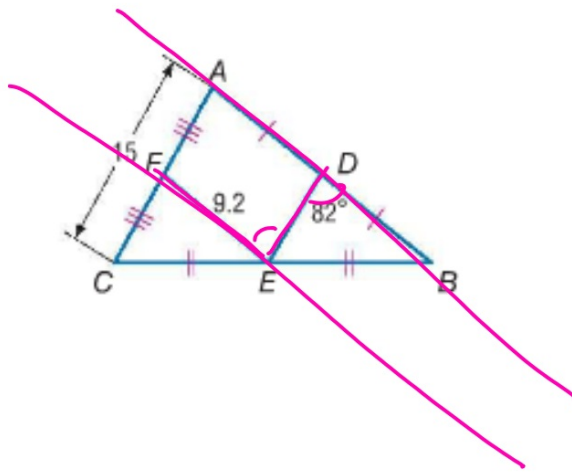
a. XZ 6.5

b. ST = 14

c. $m\angle RYX$ 124

GuidedPractice

Find each measure.

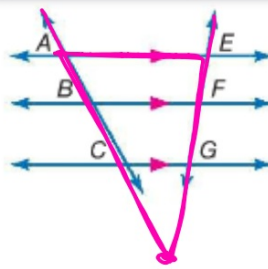**3A.** *DE*   7.5

3B. *DB*   9.2

3C. *m∠FED*   82

extend the transversals…

Corollary 7.1 Proportional Parts of Parallel Lines

If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.
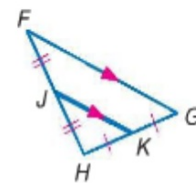
Example If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\dfrac{AB}{BC} = \dfrac{EF}{FG}$.

Theorem 7.7 Triangle Midsegment Theorem

A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.
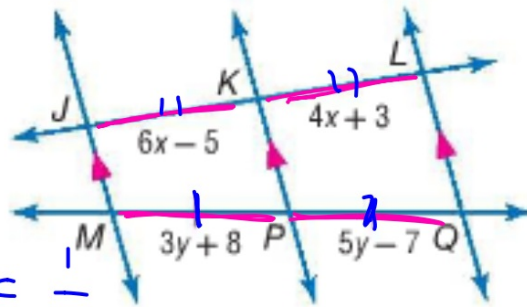
Example If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \dfrac{1}{2}FG$.

You will prove Theorem 7.7 in Exercise 32.

ALGEBRA Find x and y.

J K L
$6x - 5$ $4x + 3$
M $3y + 8$ P $5y - 7$ Q

$$\frac{6x-5}{4x+3} = \frac{1}{1} \qquad \frac{3y+8}{5y-7} = \frac{1}{1}$$

$6x - 5 = 4x + 3$ $3y + 8 = 5y - 7$

$2x = 8$ $15 = 2y$ $y = 7.5$

$x = 4$

5A.

7x − 2   8

4x + 3   8
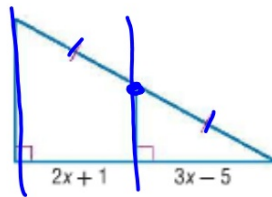
$$\frac{7x-2}{4x+3} = \frac{8}{8}$$

5B.

2x + 1   3x − 5

$$\frac{3x-5}{2x+1} = \frac{1}{1}$$

7. 4

11-430