

Geometry 8.2

Use the pythagorean theorem\*

\*8th grade standard

Use the converse of the pythagorean theorem

Prove the pythagorean theorem

leg

hypotenuse

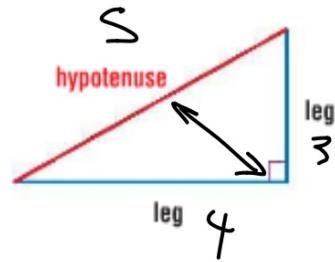
converse

~~integer~~ whole numbers

pythagorean triple ↗

$$\text{leg}^2 + \text{leg}^2 = \text{hyp}^2$$

$$(3)^2 + (4)^2 = (5)^2$$
$$9 + 16 =$$
$$25 =$$

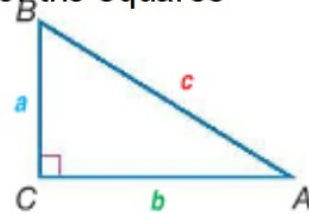


### Theorem 8.4 Pythagorean Theorem

**Words** In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

**Symbols** If  $\triangle ABC$  is a right triangle with right angle  $C$ , then  $a^2 + b^2 = c^2$ .

Sum of the squares



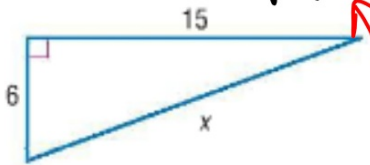
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**Example 1 Find Missing Measu**

Find x.

a.



$$6^2 + 15^2 = x^2$$

$$36 + 225 = x^2$$

$$\sqrt{261} = \sqrt{x^2}$$

Exact value or round off?

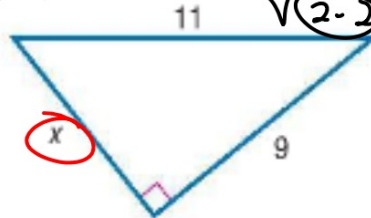
$$\sqrt{261}$$

$$\begin{array}{r} 261 \\ \wedge \\ 3 \ 87 \\ \wedge \\ 3 \ 29 \end{array}$$

$$\sqrt{3 \cdot 3 \cdot 29}$$

$$3\sqrt{29}$$

b.



$$x^2 + 9^2 = 11^2$$

$$x^2 + 81 = 121$$

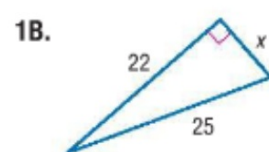
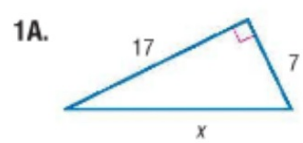
$$\sqrt{x^2} = \sqrt{40}$$

$$x = \sqrt{40}$$

$$x = 2\sqrt{10}$$

$$\sqrt{2 \cdot 2 \cdot 2 \cdot 5}$$

**Guided** Practice





**StudyTip**

**Pythagorean Triples**

If the measures of the sides of any right triangle are *not* whole numbers, the measures do not form a Pythagorean triple.

P 5 4 8

$$3^2 + 4^2 = 5^2$$

$$5^2 + 12^2 = 13^2$$

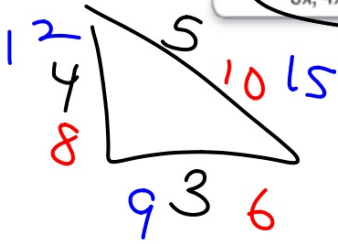
$$8^2 + 15^2 = 17^2$$

primitive (parent)  
scale factor...  
must be whole numbers

**Key Concept** Common Pythagorean Triples

3, 4, 5	5, 12, 13	8, 15, 17	7, 24, 25
x2 6, 8, 10	x2 10, 24, 26	16, 30, 34	14, 48, 50
x3 9, 12, 15	x3 15, 36, 39	24, 45, 51	21, 72, 75
3x, 4x, 5x	5x, 12x, 13x	8x, 15x, 17x	7x, 24x, 25x

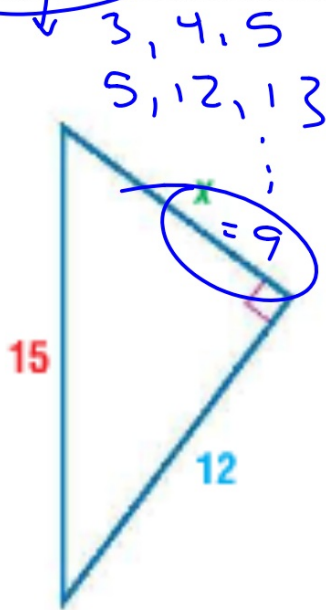
The largest number in each triple is the length of the hypotenuse.



$$7^2 + 24^2 = 25^2$$

Example 2 Use a Pythagorean Triple

Use a Pythagorean triple to find  $x$ . Explain your reasoning.



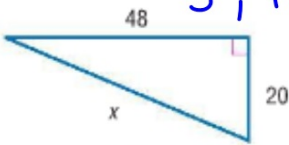
Handwritten work showing the derivation of  $x = 9$  using the 3-4-5 Pythagorean triple:

$$x, \frac{12}{3}, \frac{15}{3}$$
$$3, 4, 5$$

Maybe it is a PT. Try factoring out a GCF

Try dividing out GCF to find the primitive (parent)...

2A.



$$\frac{20}{4}, \frac{48}{4}, x$$
$$5, 12, 13$$

52

2B.



$$\frac{14}{2}, x, \frac{50}{2}$$

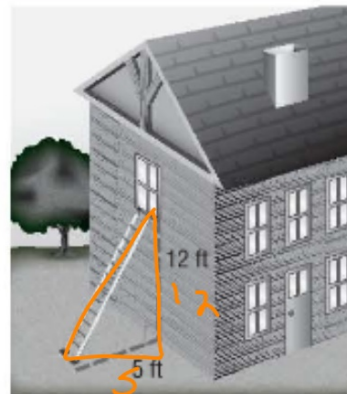
$$7, 24, 25$$

**Standardized Test Example 3** Use the Pythagorean Theorem



Damon is locked out of his house. The only open window is on the second floor, which is 12 feet above the ground. He needs to borrow a ladder from his neighbor. If he must place the ladder 5 feet from the house to avoid some bushes, what length of ladder does Damon need?

- A 7 feet
- C 13 feet**
- B 11 feet
- D 17 feet



Note: Not drawn to scale

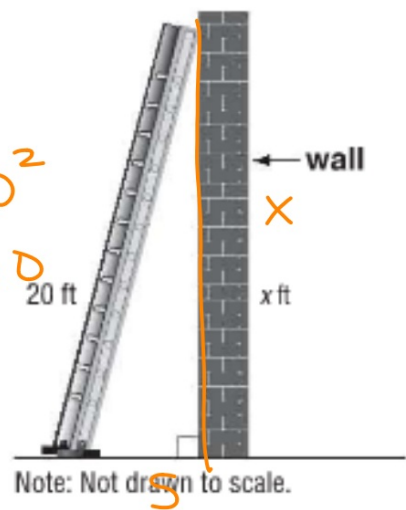


**Guided Practice**

$$\frac{S}{s}, x, \frac{20}{s}$$

3. According to your company's safety regulations, the distance from the base of a ladder to a wall that it leans against should be at least one fourth of the ladder's total length. You are given a 20-foot ladder to place against a wall at a job site. If you follow the company's safety regulations, what is the maximum distance  $x$  up the wall the ladder will reach, to the nearest tenth?

$$1, 4$$
$$S^2 + x^2 = 20^2$$
$$2S + x^2 = 400$$
$$x^2 = 37S$$



F 12 feet

H 20.6 feet

G 19.4 feet

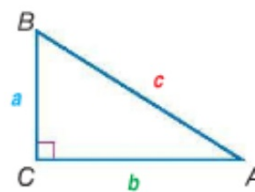
J 30.6 feet

**2 Converse of the Pythagorean Theorem** The converse of the Pythagorean Theorem also holds. You can use this theorem to help you determine whether a triangle is a right triangle given the measures of all three sides.

**Theorem 8.5** Converse of the Pythagorean Theorem

**Words** If the sum of the squares of the lengths of the shortest sides of a triangle is equal to the square of the length of the longest side, then the triangle is a right triangle.

**Symbols** If  $a^2 + b^2 = c^2$ , then  $\triangle ABC$  is a right triangle.



You will prove Theorem 8.5 in Exercise 35.

p. 550

You can also use side lengths to classify a triangle as acute or obtuse.

### StudyTip

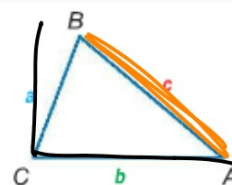
#### Determining the Longest Side

If the measures of any of the sides of a triangle are expressed as radicals, you may wish to use a calculator to determine which length is the longest.

### Theorems Pythagorean Inequality Theorems

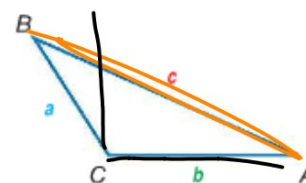
**8.6** If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, then the triangle is an acute triangle.

**Symbols** If  $c^2 < a^2 + b^2$ , then  $\triangle ABC$  is acute.



**8.7** If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, then the triangle is an obtuse triangle.

**Symbols** If  $c^2 > a^2 + b^2$ , then  $\triangle ABC$  is obtuse.



You will prove Theorems 8.6 and 8.7 in Exercises 36 and 37, respectively.

$a^2 + b^2 = (\text{perfect})$   
↑  
perfect

$a^2 + b^2 =$  perfect amount: right triangle

If longest side is less than perfect: acute

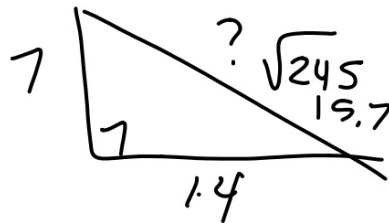
If longest side is more than perfect: obtuse

Is it a triangle at all?  
What kind of triangle?

#### Example 4 Classify Triangles

Determine whether each set of numbers can be the measures of the sides of a triangle. If so, classify the triangle as *acute*, *right*, or *obtuse*. Justify your answer.

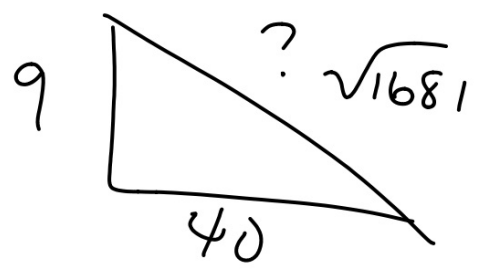
a.  $7, 14, 16$   
 $\underbrace{7 + 14}_{21} =$



b. 9, 40, 41



Right



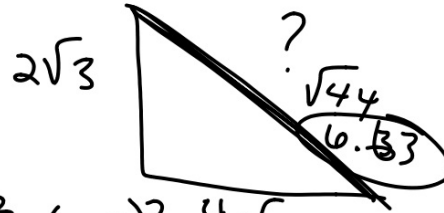
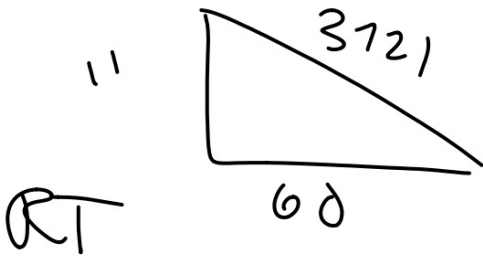
Guided Practice

4A. 11, 60, 61

3.46      5.66      (6.71)  
 ↓            ↓            ↓  
 4B.  $2\sqrt{3}, 4\sqrt{2}, 3\sqrt{5}$   
obtuse

4C. 6.2, 13.8, 20

no  $\Delta$



$$\begin{aligned} & (2\sqrt{3})^2 + (4\sqrt{2})^2 = 4\sqrt{2} \\ & 2\sqrt{3} \cdot 2\sqrt{3} + 4\sqrt{2} \cdot 4\sqrt{2} \\ & 4 \cdot 9 + 16\sqrt{4} \\ & 12 + 32 \\ & 44 \end{aligned}$$