Geometry 6.3

Recognize conditions that ensure a quadrilateral is a parallelogram

Prove that a set of points forms a parallelogram

converse

parallelogram properties

coordinate proof

midpoint formula

distance formula

slope formula

6.2 if par → prop.

6.3 if prop → par

$\left(\frac{x+x}{2}, \frac{y+y}{2}\right)$

$\frac{\text{rise}}{\text{run}}$

PT =

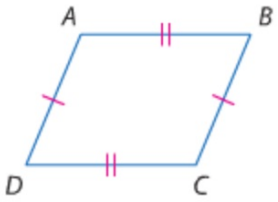
Converse...

Theorems Conditions for Parallelograms

6.9 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. sides are ≅, then quad. is a ▱.*

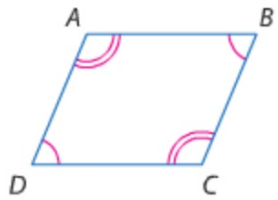
Example If $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$, then *ABCD* is a parallelogram.



6.10 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Abbreviation *If both pairs of opp. ∠s are ≅, then quad. is a ▱.*

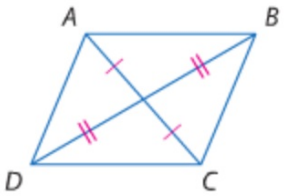
Example If $\angle A \cong \angle C$ and $\angle B \cong \angle D$, then *ABCD* is a parallelogram.



6.11 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Abbreviation *If diag. bisect each other, then quad. is a ▱.*

Example If $\overline{AC}$ and $\overline{DB}$ bisect each other, then *ABCD* is a parallelogram.



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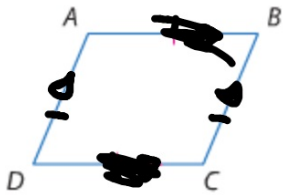
6.12 If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Abbreviation *If one pair of opp. sides is ≅ and ||, then the quad. is a ▱.*

Example If $\overline{AB} \parallel \overline{DC}$ and $\overline{AB} \cong \overline{DC}$, then *ABCD* is a parallelogram.



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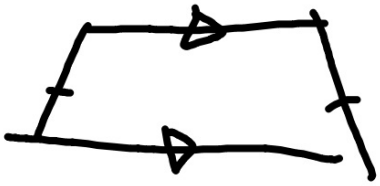
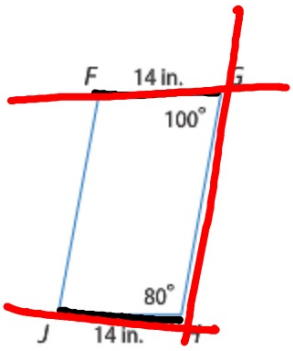


Can't go by eyeball... which theorem/property applies?

Example 1 Identify Parallelograms

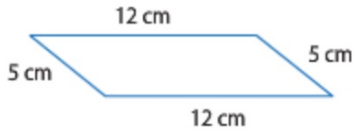
Determine whether the quadrilateral is a parallelogram. Justify your answer.



1 pr opp ≅ & ∥

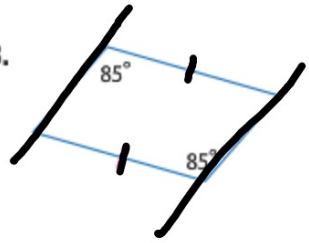
GuidedPractice

1A.



2 prs ≅ opp sides

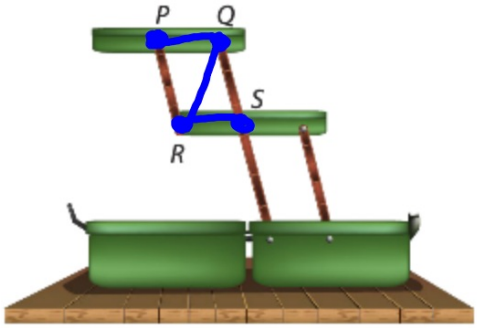
1B.



no

Real-World Example 2 Use Parallelograms to Prove

FISHING The diagram shows a side view of the tackle box at the left. In the diagram, $PQ = RS$ and $PR = QS$. Explain why the upper and middle trays remain parallel no matter to what height the trays are raised or lowered.



▱ PQSR

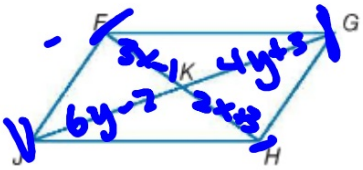
Real-World Link

A 2- or 3-cantilever tackle box is often used to organize lures and other fishing supplies. The trays lift up and away so that all items in the box are easily accessible.

If I want it to be a parallelogram, what has to be true?

$x = 4$

$y = 2.5$



Example 3 Use Parallelograms and Algebra to Find Value

If $FK = 3x - 1$, $KG = 4y + 3$, $JK = 6y - 2$, and $KH = 2x + 3$, find x and y so that the quadrilateral is a parallelogram.

$FK = KH$

$3x - 1 = 2x + 3$

$-2x + 1 \quad -2x + 1$

$x = 4$

$JK = KG$

$6y - 2 = 4y + 3$

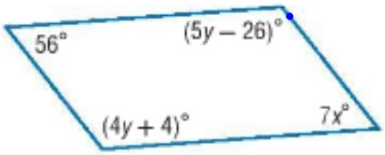
$-4y + 2 \quad -4y + 2$

$2y = 5 \quad y = 2.5$

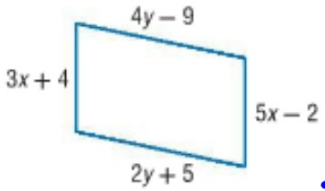
GuidedPractice

Find x and y so that each quadrilateral is a parallelogram.

3A.



3B.



$4y-9=2y+5$

$3x+4=5x-2$

$\frac{56}{7}=\frac{7x}{7}$

$8=x$

$4y+4=5y-26$

$-4y+26 \quad -4y+26$

$30=y$

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Concept Summary

Prove that a Quadrilateral Is a Parallelogram

- Show that both pairs of opposite sides are parallel. (Definition)
- Show that both pairs of opposite sides are congruent. (Theorem 6.9)
- Show that both pairs of opposite angles are congruent. (Theorem 6.10)
- Show that the diagonals bisect each other. (Theorem 6.11)
- Show that a pair of opposite sides is both parallel and congruent. (Theorem 6.12)

StudyTip

Midpoint Formula

To show that a quadrilateral is a parallelogram, you can also use the Midpoint Formula. If the midpoint of each diagonal is the same point, then the diagonals bisect each other.

GuidedPractice

Determine whether the quadrilateral is a parallelogram. Justify your answer using the given formula.

4A. $A(3, 3), B(8, 2), C(6, -1), D(1, 0)$ Dista

What properties of parallogram depend on distance (lengths)?

$\overline{AB} \cong \overline{DC}$

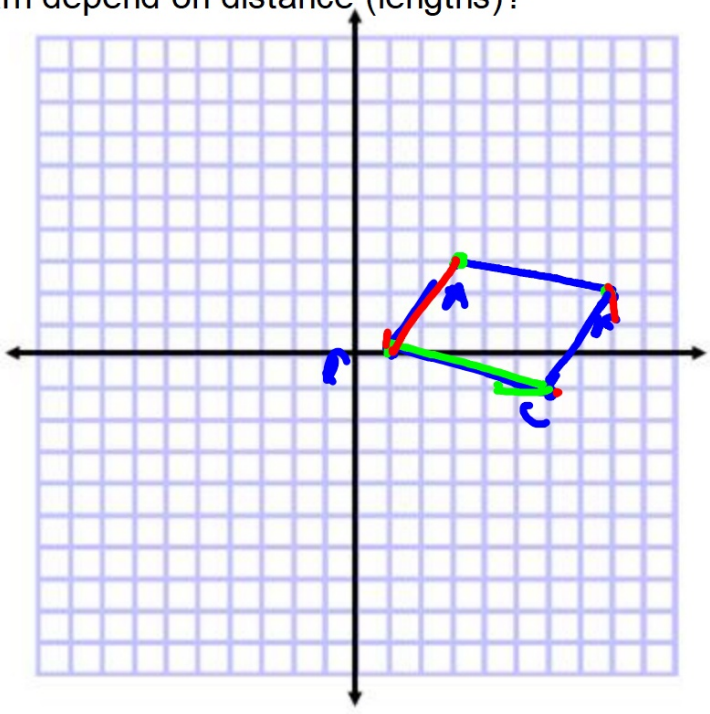
$\sqrt{5^2+1^2} = \sqrt{5^2+1^2}$

$\sqrt{26} = \sqrt{26}$

$\overline{DA} \cong \overline{BC}$

$\sqrt{3^2+2^2} = \sqrt{3^2+2^2}$

$\sqrt{13} = \sqrt{13}$



4B. $F(-2, 4)$, $G(4, 2)$, $H(4, -2)$, $J(-2, -1)$; Midpoint Formula

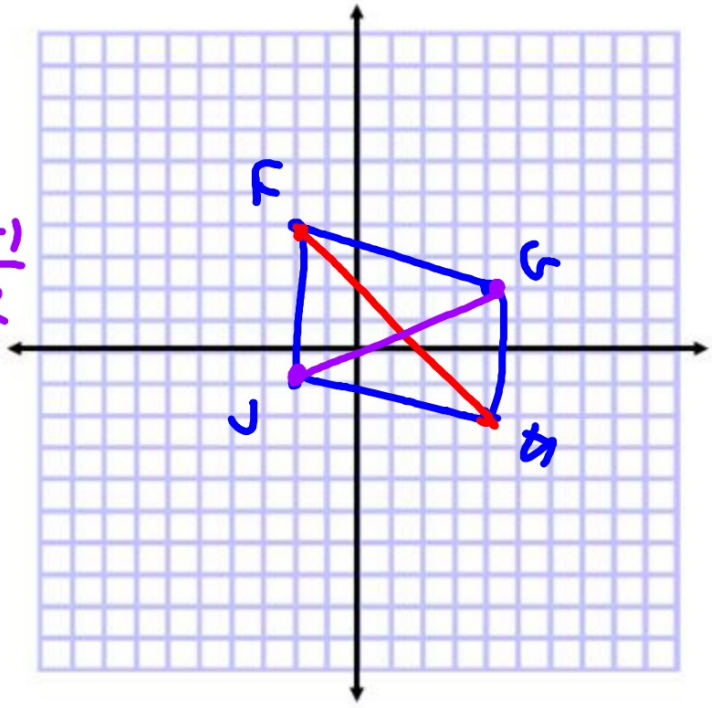
What parallelogram properties depend on midpoints?

mp of diag.

mp FH mp JG

$\frac{-2+4}{2}$ $\frac{4+-2}{2}$ $\frac{4+-2}{2}$ $\frac{2+-1}{2}$

$(1, 1)$ $(1, \frac{1}{2})$



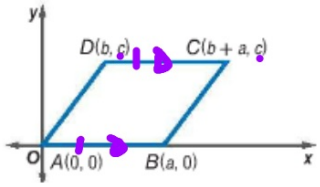
use slopes

Step 2 Use your figure to write a proof.

Given: quadrilateral $ABCD$, $\overline{AB} \parallel \overline{DC}$, $\overline{AB} \cong \overline{DC}$

Prove: $ABCD$ is a parallelogram.

Coordinate Proof:



Hint: Prove that it has 2 pairs of parallel sides

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1. $\overline{AB} \parallel \overline{DC}$ $AB \cong DC$	1. given
2. slope CD = $\frac{0}{a}$ slope AB = $\frac{0}{a}$	2. parallel (same slope)
3. slope AD = $\frac{c}{b}$ slope BC = $\frac{c}{b}$	3.
4. ABCD is □	4. def □