

Geometry 6.1

Find and use the sum of the measures of the interior angles of a polygon.

Find and use the sum the of the exterior angles of a polygon

polygon

convex

concave

triangle sum theorem

*consecutive ^{in a row}

nonconsecutive ^{not in a row}

diagonal - connects ^{non-conv. vert.}

interior angles (of a polygon)

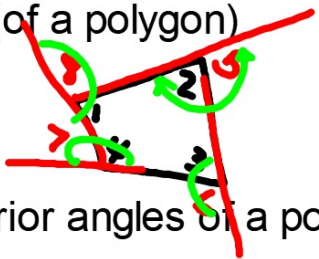
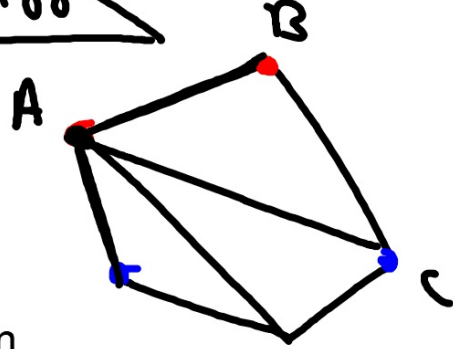
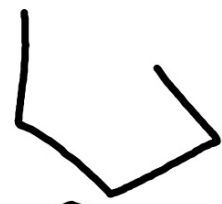
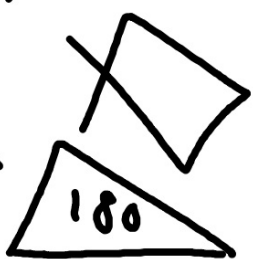
exterior angles (of a polygon)

regular polygon

polygon names

exploration: interior angles of a polygon

straight sides; meet @ ends; closed



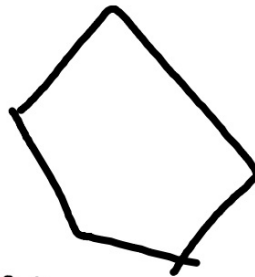
StudyTip

Naming Polygons

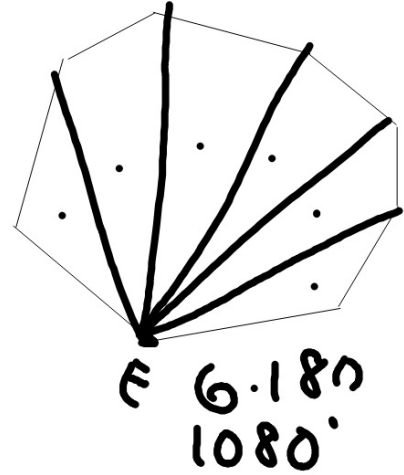
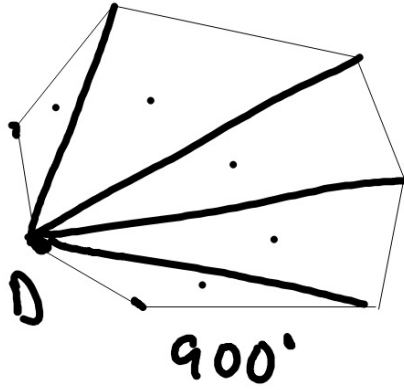
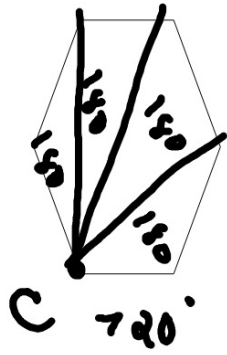
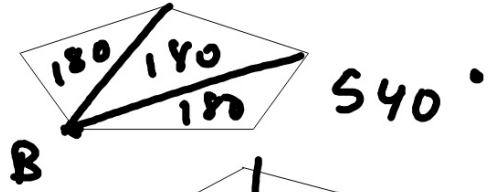
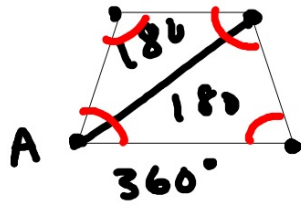
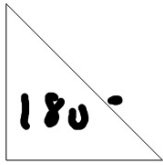
Remember, a polygon with n -sides is an n -gon, but several polygons have special names.

| Number of Sides | Polygon |
|-----------------|-----------------------|
| 3 | triangle |
| 4 | quadrilateral |
| 5 | pentagon |
| 6 | hexagon |
| → 7 | heptagon |
| 8 | octagon |
| → 9 | nonagon |
| 10 | decagon |
| 11 | hendecagon |
| 12 | dodecagon |
| n | n -gon |

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11-gon
12-gon



| | n | $n-2$ | |
|---------------|-----------------|---------------------|---|
| Polygon | Number of sides | Number of triangles | Sum of interior angle measures <i>degrees</i> |
| Triangle | 3 | 1 | 180° |
| Quadrilateral | 4 | 2 | 360° |
| Pentagon | 5 | 3 | 540° |
| Hexagon | 6 | 4 | 720° |
| Heptagon | 7 | 5 | 900° |
| Octagon | 8 | 6 | 1080° |

What do you notice?
What do you wonder?

Decagon

n-gon

10

n

8

$n-2$

$8 \cdot 180 = 1440$

$(n-2)(180)$

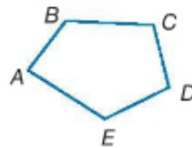
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Don't memorize...
How many triangles can you form
(from a vertex)?

Theorem 6.1 Polygon Interior Angles Sum

The sum of the interior angle measures of an n -sided convex polygon is $(n - 2) \cdot 180$.

Example $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E = (5 - 2) \cdot 180$
 $= 540$



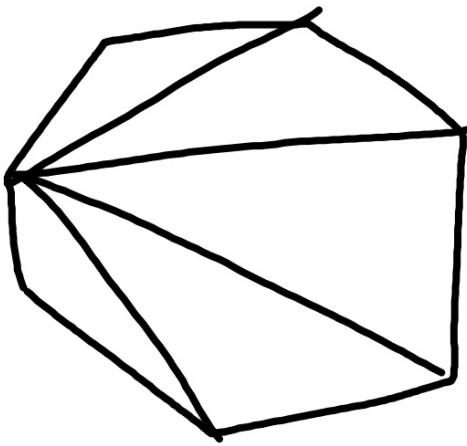
You will prove Theorem 6.1 for octagons in Exercise 42.



Example 1 Find the Interior Angles Sum of a Polygon

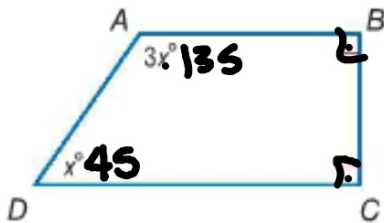


- a. Find the sum of the measures of the interior angles of a convex heptagon.



$$5 \cdot 180 = 900$$

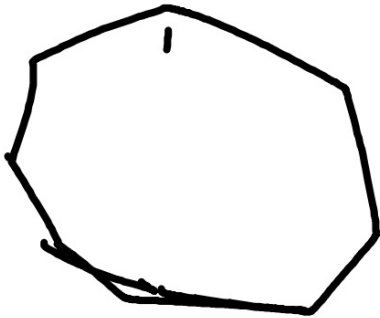
- b. **ALGEBRA** Find the measure of each interior angle of quadrilateral $ABCD$.



$$\begin{array}{r} 180 + 4x = 360 \\ -180 \qquad -180 \\ \hline 4x = 180 \\ \frac{4}{4} \qquad \frac{180}{4} \\ x = 45 \end{array}$$

Guided Practice

1A. Find the sum of the measures of the interior angles of a regular octagon.



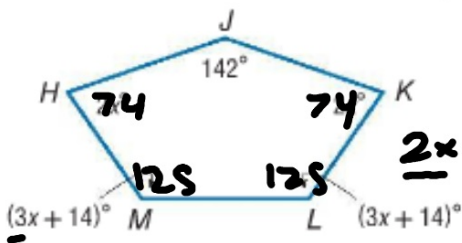
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$$6 \cdot 180 = 1080$$

$$\frac{1080}{8} = 135^\circ$$

regular
all sides \cong
all \angle s \cong

1B. Find the measure of each interior angle of pentagon $HJKLM$ shown



$$3(180)$$

$$\underline{2x} + 142 + \underline{2x} + \underline{3x} + 14 + \underline{3x} + 14 = 540$$

$$10x + 170 = 540$$

$$10x = 370$$

$$x = 37$$

regular:

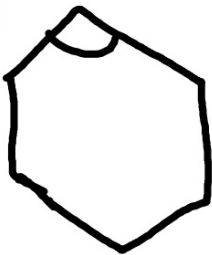
Review Vocabulary

regular polygon

a convex polygon in which all of the sides are congruent and all of the angles are congruent

Real-World Example 2 Interior Angle Measure of Regular Polygon

TENTS The poles for a tent form the vertices of a regular hexagon. When the poles are properly positioned, what is the measure of the angle formed at a corner of the tent?



$$4 \cdot 180 = 720$$

$$120$$

Guided Practice

2A. **COINS** Find the measure of each interior angle of the regular ~~hendecagon~~ **11-gon** that appears on the face of a Susan B. Anthony one-dollar coin.

hendecagon (11 sides)

$$9 \cdot 180 = 1620$$

$$n = 147.3^\circ$$

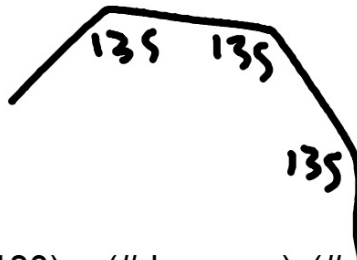
28. **HOT TUBS** A certain company makes hot tubs in a variety of different shapes. Find the measure of each interior angle of the nonagon model.
-

IF regular polygon... $\text{Sum exterior} = 360$

Example 3 Find Number of Sides Given Interior Angle Measure



The measure of an interior angle of a regular polygon is 135. Find the number of sides in the polygon.



$$(\# \text{triangles}) \times (180) = (\# \text{degrees}) \times (\# \text{angles})$$

$$n = 6$$

$$180 \cdot n = 135(n+2)$$

$$180n = 135n + 270$$

$$\begin{array}{r} -135n \\ \hline \end{array}$$

$$45n = 270$$

