Geometry 13.4
Design simulations to estimate probabilities
Summarize data from simulations

probability model
simulation $n \geq 30$ assumptions:
random
expected value
Law of large numbers

1 Design a Simulation A **probability model** is a mathematical model used to match a random phenomenon. A **simulation** is the use of a probability model to recreate a situation again and again so that the likelihood of various outcomes can be estimated. To design a simulation, use the following steps.

p. 939

KeyConcept Designing a Simulation

Step 1 Determine each possible outcome and its theoretical probability.

Step 2 State any assumptions.

Step 3 Describe an appropriate probability model for the situation.

Must have the same probabilities as your situation

Step 4 Define what a trial is for the situation and state the number of trials to be conducted.

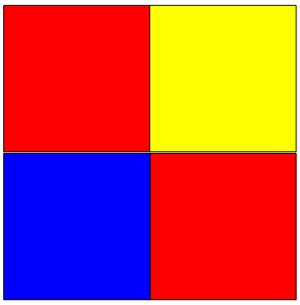
Minimum #trials 30

Model:
spinner
dice
random number
flip coin
etc.

2. SOCCER Last season, Yao made 18% of his free kicks. Design a simulation using a random number generator that can be used to estimate the probability that he will make his next free kick.

Use 100 slips of paper...

Outcome make / miss
Assumptions
Model
Trial



Dart board:
Expected value:
Red = 1 point
Yellow = 4 points
Blue = 6 points

$EV =$

$$\text{Red}\ (.5)(1) = 0.5$$
$$\text{Y}\ \ (.25)(4) = 1$$
$$\text{B}\ \ (.25)(6) = 1.5$$
$$\overline{\qquad\qquad\qquad\qquad} \div 3$$

Expected value:
Over the course of multiple rounds (n>30)
Average amount expected on a typical turn...

Expected value:
Roll number cube:
1 = 4 pts
even number = 10 pts
3 = 1 pt
5 = 2 pts

$$
\begin{aligned}
1 \quad & \tfrac{1}{6} \cdot 4 = \tfrac{4}{6} \\
2 \quad & \tfrac{1}{6} \cdot 10 = \tfrac{10}{6} \\
3 \quad & \tfrac{1}{6} \cdot 1 = \tfrac{1}{6} \\
4 \quad & \tfrac{1}{6} \cdot 10 = \tfrac{10}{6} \\
5 \quad & \tfrac{1}{6} \cdot 2 = \tfrac{2}{6} \\
6 \quad & \tfrac{1}{6} \cdot 10 = \tfrac{10}{6} \\
& \hline \\
& \tfrac{37}{6}
\end{aligned}
$$

$EV \approx 6.2$

Average amount expected on a typical turn...(overall)

9.941

Expected value, also known as mathematical expectation, is the average value of a random variable that one *expects* after repeating an experiment or simulation a theoretically infinite number of times. To find the expected value $E(X)$ of a random variable X, follow these steps.

"point value"

KeyConcept Calculating Expected Value

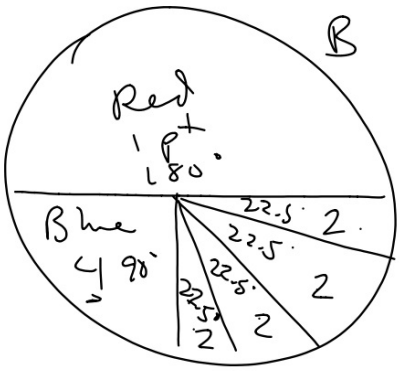
Step 1 Multiply the value of X by its probability of occurring.

Step 2 Repeat Step 1 for all possible values of X.

Step 3 Find the sum of the results.

=

overall average if you play multiple rounds...



R $0.5 \cdot 1 = 0,5$

B $0,25 \cdot 4 = 1$

a $0,0625 \cdot 2 = 0,125$

b $0,0625 \cdot 2 = 0,125$

c $0,0625 \cdot 2 = 0,125$

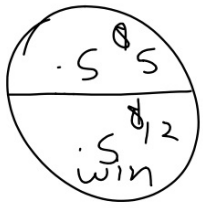
d $0,0625 \cdot 2 = 0,125$

$EV = 2$

Fair ? Not Fair ?

$1 ↓ $5

Compare EV & prize



$.5(5) = 2.5$

$.5(12) = 6$

$8.5