Geometry   10.6
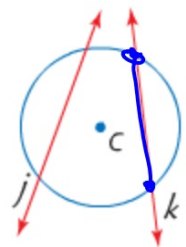Find measures of ==angles== formed by lines intersecting:
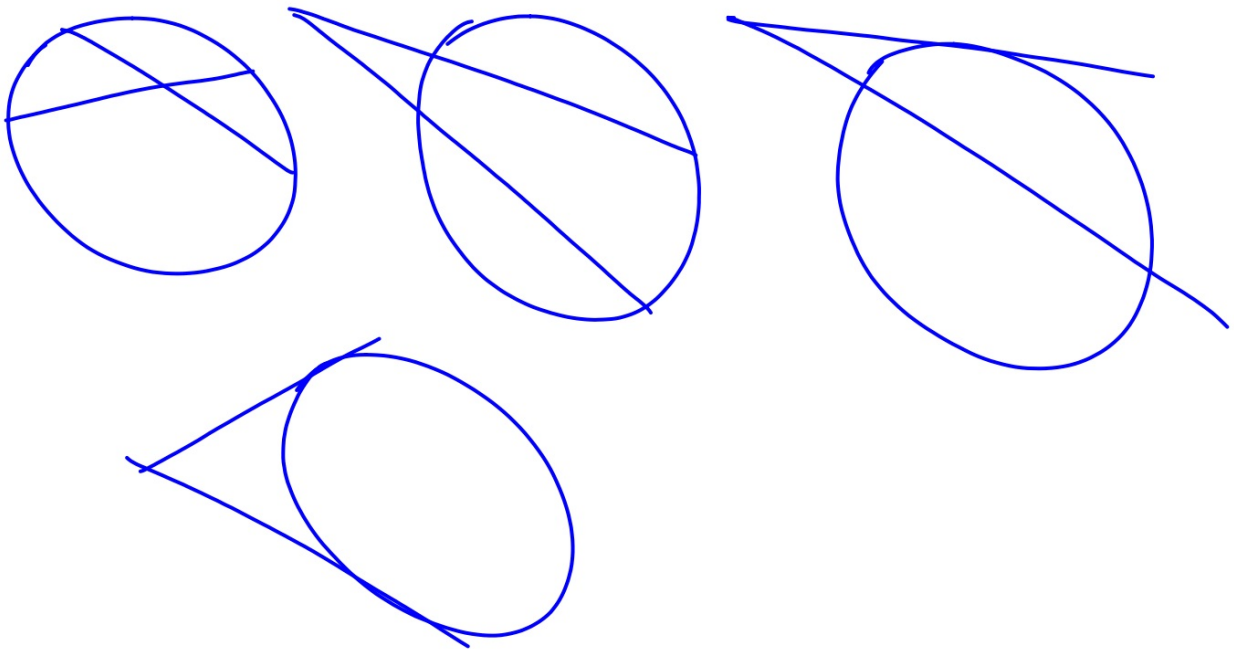...inside the circle
... outside the circle

tangent line line touches
secant line   intersect 2x
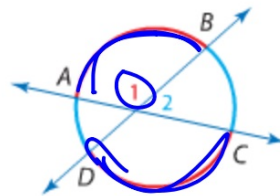hula hoops & meter sticks

Circle + 2 intersecting lines:

Not a central angle...
But they are vertical angles!

$$angle = \frac{1}{2}\left(\underset{arcs}{Sum}\right)$$

Theorem 10.12

Words   If two secants or chords intersect in the
interior of a circle, then the measure of an
angle formed is one half the *sum* of the
measure of the arcs intercepted by the angle
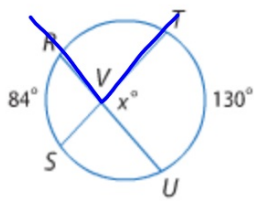and its vertical angle.

Example $m\angle 1 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$ and $m\angle 2 = \frac{1}{2}(m\widehat{DA} + m\widehat{BC})$

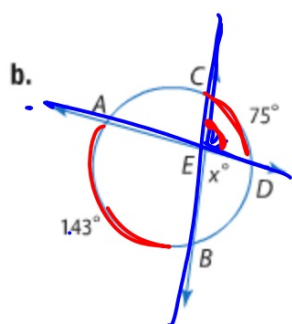$$= \frac{1}{2}\left(\widehat{AB} + \widehat{CD}\right)$$

Example 1 Use Intersec

Find x.

a.

$84°$ $130°$

R T V $x°$ S U

angle $= \frac{1}{2}(\text{sum})$

$x = \frac{1}{2}(130 + 84)$

$x = 107$

b.

75°

1.43°

$x = \frac{1}{2} (\qquad)$

$x = 71°$

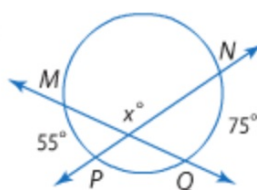$y = \frac{1}{2}(143+75)$

$y = 109$

Which angle is x?

GuidedPractice

1A.

H 47°
G
x°
L
K
116°

1B.

M
N
x°
55°
75°
P   Q

1C.

154°   Y
128
W   Z
X   x°

$x = \frac{1}{2}(\qquad)$
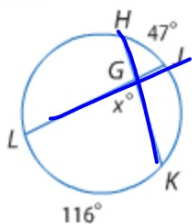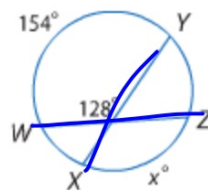
$128 = \frac{1}{2}(154 + x)$

$128 = 77 + \frac{1}{2}x$

$51 = \frac{1}{2}x$

$x = 102°$

(same as inscribed angle)

Theorem 10.13

Words If a secant and a tangent intersect at the point of tangency, then the measure of each angle formed is one half the measure of its intercepted arc.
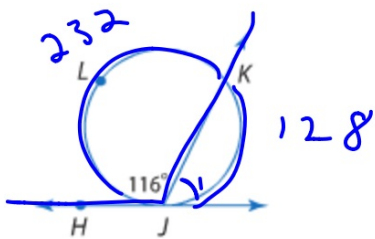
Example $m\angle 1 = \dfrac{1}{2}m\widehat{AB}$ and $m\angle 2 = \dfrac{1}{2}m\widehat{ACB}$
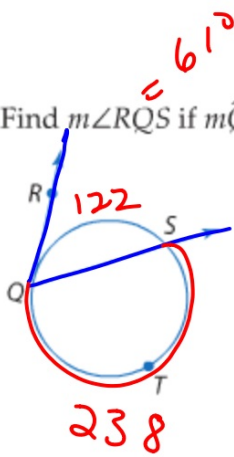
80

6 2

B

C

A
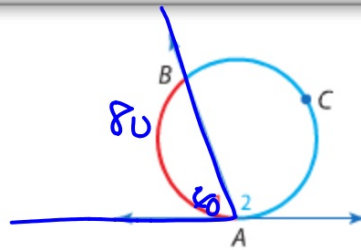
You will prove Theorem 10.13 in Exercise 33.

2A. Find $m\widehat{JLK}$.

232

128

116°

H   J

L   K

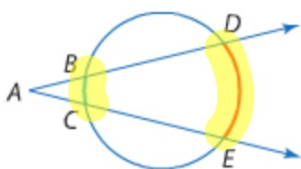**2B.** Find $m\angle RQS$ if $m\widehat{QTS} = 238$.

$= 61°$

122

238

Intersection outside the circle...turns into 1/2 the **difference** of

Theorem 10.14

Words If two secants, a secant and a tangent, or two tangents intersect in the exterior of a circle, then the measure of the angle formed is one half the *difference* of the measures of the intercepted arcs.
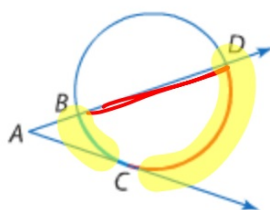
$$ange = \frac{1}{2}(\ diff\ )$$

Examples

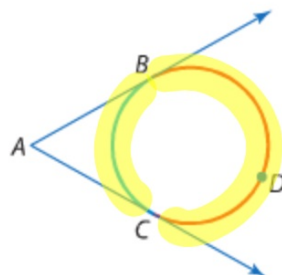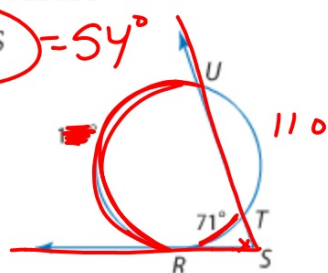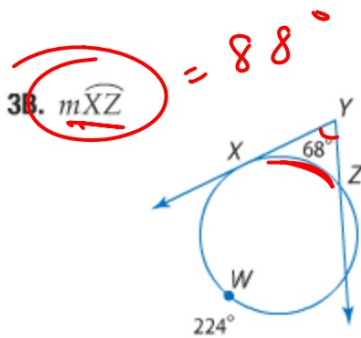| Two Secants	Secant-Tangent	Two Tangents
$m\angle A = \frac{1}{2}(m\widehat{DE} - m\widehat{BC})$	$m\angle A = \frac{1}{2}(m\widehat{DC} - m\widehat{BC})$	$m\angle A = \frac{1}{2}(m\widehat{BDC} - m\widehat{BC})$

3A. $m\angle S$ = 54°

U

110

71° T

R   S

$x = \frac{1}{2}(179 - 71)$

$\overgroup{} = 88°$

3B. $m\widehat{XZ}$

Y
X 68°
Z
W
224°
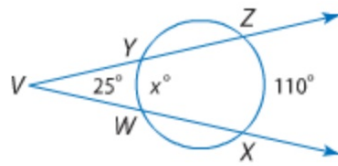
$$68 = \frac{1}{2}(224 - n)$$
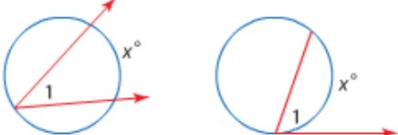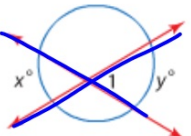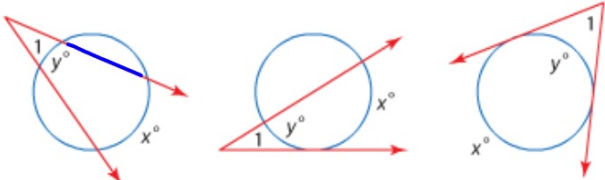$$68 = 112 - \frac{1}{2}n$$
$$-44 = -\frac{1}{2}n \qquad n = 88$$

4. Find the value of x.

$$25 = \frac{1}{2}(110 - x)$$

KeyConcept Circle and Angle Relationships

Vertex of Angle	Model(s)	Angle Measure
on the circle		one half the measure of the intercepted arc $m\angle 1 = \frac{1}{2}x$
inside the circle		one half the measure of the sum of the intercepted arc $m\angle 1 = \frac{1}{2}(x + y)$
outside the circle		one half the measure of the difference of the intercepted arcs $m\angle 1 = \frac{1}{2}(x - y)$

p745

stupid Kroon trick (?)

WB 10.6