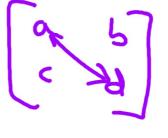
Algebra 2 3.8
Find the inverse of a 2x2 matrix
Write and solve matrix equations

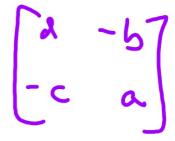
square matrix identity matrix inverse matrix matrix equation coefficient matrix variable matrix constant matrix whiteboards

Wife Swap (TV?

Matrix equation is different than Cramer's rule...

(Most students have a favorite after they try both methods.)



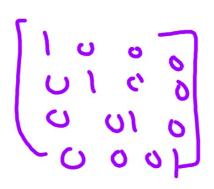


2 × 2 Identity Matrix

3 × 3 Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$





KeyConcept Identity Matrix for Multiplication

Words

The identity matrix for multiplication / is a square matrix with 1 for every element of the main diagonal, from upper left to lower right, and 0 in all other positions. For any square matrix A of the same dimension as I, $A \cdot I = I \cdot A = A$.

Symbols

If
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
, then $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Two $n \times n$ matrices are inverses of each other if their product is the identity matrix. If matrix A has an inverse symbolized by A^{-1} , then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

$$A \cdot A^{-1} = I$$

Example 1 Verify Inverse Matrices

Determine whether the matrices in each pair are inverses.

a.
$$A = \begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix}$$
 and $B = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix}$

$$\begin{bmatrix} -4 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Determine whether the matrices in each pair are inverses.

1.
$$A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

2.
$$C = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix} =$$

KeyConcept Inverse of a 2×2 Matrix

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

What is ad-bc? 1/det

Wife Swap

Example 2 Find the Inverse of a Matrix

Find the inverse of each matrix, if it exists.

a.
$$P = \begin{bmatrix} 7 & -5 \\ 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 5 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 3 & -2 \end{bmatrix}$$

Why would an inverse not exist?

b.
$$Q = \begin{bmatrix} -8 \\ 12 \end{bmatrix} = \begin{bmatrix} -8 \\ 9 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} -8 \\ 12 \end{bmatrix} = \begin{bmatrix} -8 \\ 9 \end{bmatrix} = \begin{bmatrix} -72 \\ 0 \end{bmatrix} = \begin{bmatrix} -72 \\ 0 \end{bmatrix}$$

whiteboards

GuidedPractice

A= [
$$\frac{7}{2}$$
] = $\frac{7}{4}$ [$\frac{2}{4}$] $\frac{1}{3}$]

A= $\frac{7}{4}$ = $\frac{7}{4}$ [$\frac{7}{4}$] $\frac{7}{3}$]

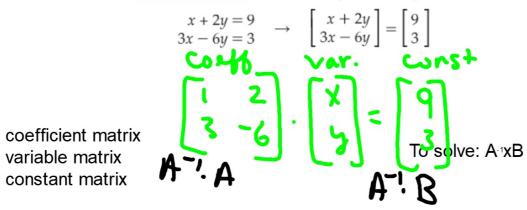
Find the inverse of each matrix, if it exists.

5.
$$\begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$$

6.
$$\begin{bmatrix} 2 & -4 \\ -3 & 0 \end{bmatrix}$$

Different than Cramer's rule...

Matrix Equations Matrices can be used to represent and solve systems of equations. You can write a matrix equation to solve the system of equations below.



Use a matrix equation to solve each system of equations.

9.
$$-2x + y = 9$$
 $x + y = 3$

$$y = 9$$
 10. $4x - 2y = 22$ $6x + 9y = -3$