Algebra 2 3.8

Find the inverse of a 2x2 matrix

Write and solve matrix equations

square matrix

identity matrix

inverse matrix

matrix equation

coefficient matrix

variable matrix

constant matrix

whiteboards

Wife Swap (TV?)

$$\begin{bmatrix} & \\ & \end{bmatrix}$$

2x2

Matrix equation is different than Cramer's rule...

(Most students have a favorite after they try both methods.)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2 × 2 Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3 × 3 Identity Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

KeyConcept Identity Matrix for Multiplication

Words The identity matrix for multiplication I is a square matrix with 1 for every element of the main diagonal, from upper left to lower right, and 0 in all other positions. For any square matrix A of the same dimension as I, $A \cdot I = I \cdot A = A$.

Symbols If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ such that

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Two $n \times n$ matrices are **inverses** of each other if their product is the identity matrix. If matrix A has an inverse symbolized by A^{-1}, then $A \cdot A^{-1} = A^{-1} \cdot A = I$.

$A \cdot A^{-1} = I$

Is their product the identity matrix?

Example 1 Verify Inverse Matrices

Determine whether the matrices in each pair are inverses.

a. $A = \begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix}$

$$\begin{bmatrix} -4 & 2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{4} & -\frac{1}{2} \\ \frac{1}{2} & -1 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$-1+1$

$$\begin{bmatrix} 0 & \\ & \end{bmatrix}$$

Determine whether the matrices in each pair are inverses.

1. $A = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

2. $C = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix}$

$$\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 5 & -3 \end{bmatrix} =$$

KeyConcept Inverse of a 2 × 2 Matrix

The inverse of matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $ad - bc \neq 0$.

$$\frac{1}{\text{det}}\begin{bmatrix} \text{Wife} \\ \text{Swap} \end{bmatrix}$$

What is ad-bc?
1/det

Wife Swap

Example 2 Find the Inverse of a Matrix

Find the inverse of each matrix, if it exists.

a. $P = \begin{bmatrix} 7 & -5 \\ 2 & -1 \end{bmatrix}$

$$\frac{1}{3}\begin{bmatrix} -1 & 5 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & \frac{5}{3} \\ -\frac{2}{3} & \frac{7}{3} \end{bmatrix}$$

Why would an inverse not exist?

-72

b. $Q = \begin{bmatrix} -8 & -6 \\ 12 & 9 \end{bmatrix}$ $\quad \frac{1}{det} \cdot \begin{bmatrix} w.s \end{bmatrix}$

-72

$Q^{-1} = \quad \frac{1}{0} \quad NP$

whiteboards

$-12-7$

Guided Practice

2A. $A = \begin{bmatrix} 3 & 7 \\ 1 & -4 \end{bmatrix} = \frac{1}{\det}\begin{bmatrix} w & s \end{bmatrix}$

-12

$A^{-1} = \frac{-1}{19}\begin{bmatrix} -4 & -7 \\ -1 & 3 \end{bmatrix}$

2B. $\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$

Find the inverse of each matrix, if it exists.

5. $\begin{bmatrix} 6 & -3 \\ -1 & 0 \end{bmatrix}$

6. $\begin{bmatrix} 2 & -4 \\ -3 & 0 \end{bmatrix}$

Different than Cramer's rule...

2 Matrix Equations Matrices can be used to represent and solve systems of equations. You can write a **matrix equation** to solve the system of equations below.

$$\begin{aligned} x + 2y &= 9 \\ 3x - 6y &= 3 \end{aligned} \quad \rightarrow \quad \begin{bmatrix} x + 2y \\ 3x - 6y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$$\underset{\text{coeff}}{\begin{bmatrix} 1 & 2 \\ 3 & -6 \end{bmatrix}} \cdot \underset{\text{var.}}{\begin{bmatrix} x \\ y \end{bmatrix}} = \underset{\text{const}}{\begin{bmatrix} 9 \\ 3 \end{bmatrix}}$$

coefficient matrix
variable matrix
constant matrix

$A^{-1} \cdot A$

To solve: $A^{-1}xB$

$A^{-1} \cdot B$

Use a matrix equation to solve each system of equations.

9. $-2x + y = 9$
   $x + y = 3$

$$\begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$$

$A^{-1} \cdot A$ $\qquad$ $A^{-1} \cdot B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \cdot B$$

10. $4x - 2y = 22$
    $6x + 9y = -3$