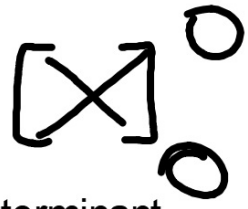
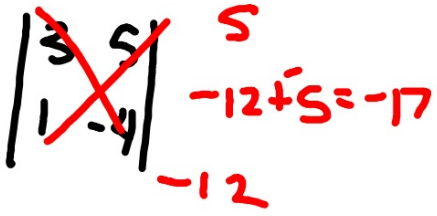
Algebra 2 3.7

Evaluate determinants

Use Cramer's rule to solve systems of equations

Quiz 3.5-3.6 Wed.





matrix

square matrix

determinant

2x2 second order determinant

3x3 third order determinant

diagonal method, cofactor method, echelon method

* Cramer's rule

coefficient matrix

whiteboards

KeyConcept Second-Order Determinant

Words The value of a second-order determinant is the difference of the products of the two diagonals.

Symbols $\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Example $\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = 4(6) - (5)(-3) = 39$

Notice how the notation has changed...
Does it look like something else?
context...

Example 1 Second-Order Determinant

Evaluate each determinant.

a. $\begin{vmatrix} 5 & -4 \\ 8 & 9 \end{vmatrix}$ -32

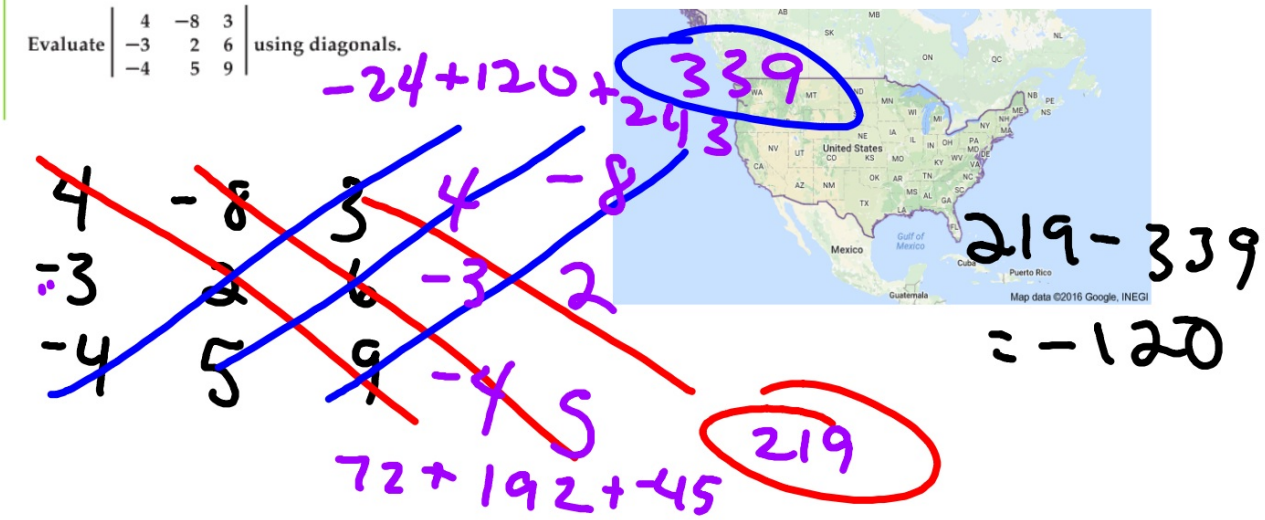
45

b. $\begin{vmatrix} 0 & 6 \\ 4 & -11 \end{vmatrix}$

There are multiple methods to evaluate a determinant for 3x3
(We are going to learn the diagonal method...for now.)

Example 2 Use Diagonals

Evaluate $\begin{vmatrix} 4 & -8 & 3 \\ -3 & 2 & 6 \\ -4 & 5 & 9 \end{vmatrix}$ using diagonals.



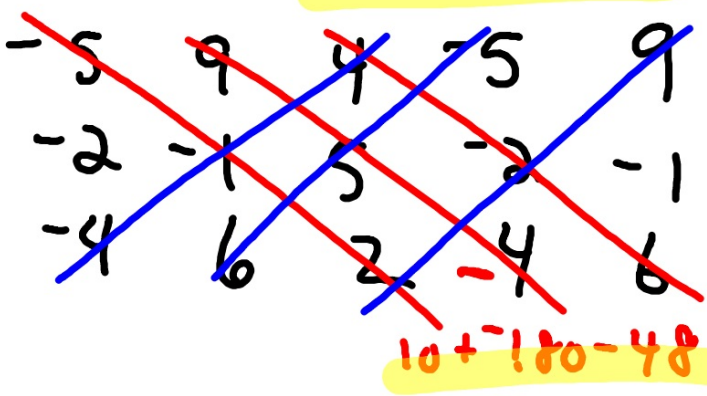
GuidedPractice

Evaluate each determinant.

2A. $\begin{vmatrix} -5 & 9 & 4 \\ -2 & -1 & 5 \\ -4 & 6 & 2 \end{vmatrix}$

2B. $\begin{vmatrix} -8 & -4 & 4 \\ 0 & -5 & -8 \\ 3 & 4 & 1 \end{vmatrix}$

16 -150 -36 = -170



10 + -180 - 48 = -218

-218 - -170

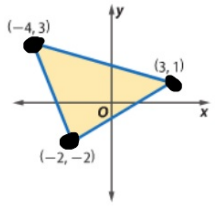
-218 + 170

= -48

KeyConcept Area of a Triangle

Words The area of a triangle with vertices (a, b), (c, d), and (e, f) is $|A|$, where

$$A = \frac{1}{2}\begin{vmatrix} a & b & 1 \\ c & d & 1 \\ e & f & 1 \end{vmatrix}.$$



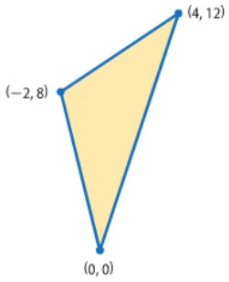
Example

$$A = \frac{1}{2}\begin{vmatrix} -4 & 3 & 1 \\ 3 & 1 & 1 \\ -2 & -2 & 1 \end{vmatrix}$$

$A = \frac{1}{2}bh.$

p 195
27-43 o

$$\frac{1}{2}\begin{vmatrix} -4 & 3 & 1 \\ 3 & 1 & 1 \\ -2 & -2 & 1 \end{vmatrix}$$



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KeyConcept Cramer's Rule

Let C be the coefficient matrix of the system $ax + by = m$, $fx + gy = n$ $\rightarrow \begin{bmatrix} a & b \\ f & g \end{bmatrix}$.

The solution of this system is $x = \dfrac{\begin{vmatrix} m & b \\ n & g \end{vmatrix}}{|C|}$ and $y = \dfrac{\begin{vmatrix} a & m \\ f & n \end{vmatrix}}{|C|}$, if $|C| \neq 0$.

coefficient matrix
variable matrix (either x or y)

$x = \dfrac{|\text{coef matrix}|}{|\text{coef matrix}|}$ $\quad y = \dfrac{|\text{coef matrix}|}{|\text{coef matrix}|}$

Example 4 Solve a System of Two Equations

Solve the system by using Cramer's Rule.

$5x - 6y = 15$
$3x + 4y = -29$

$$x = \begin{vmatrix} 15 & -6 \\ -29 & 4 \end{vmatrix} \Big| \begin{matrix} 174 \\ 60 \end{matrix} \quad \begin{matrix} \\ 60-174 \end{matrix}$$

$$\begin{vmatrix} 5 & -6 \\ 3 & 4 \end{vmatrix} \Big| \begin{matrix} -18 \\ 20+18 \end{matrix} \quad \begin{matrix} \\ 20 \quad 45 \end{matrix}$$

$$y = \begin{vmatrix} 5 & 15 \\ 3 & -29 \end{vmatrix}$$

-145
$-145-45$
-190

Test-Taking Tip

Cramer's Rule When the determinant of the coefficient matrix C is 0, the system does not have a unique solution.

$\left(\frac{-114}{38}, \frac{-190}{38}\right)$ $(-3, -5)$

Zero in the denominator...

$$7x + 3y = 37$$
$$-5x - 7y = -41$$

$$x = \begin{vmatrix} 37 & 3 \\ -41 & -7 \end{vmatrix} \begin{matrix} -123 \\ -259 \end{matrix} \quad \begin{matrix} -259 + 123 \\ = -136 \end{matrix}$$

$$y = \begin{vmatrix} 7 & 37 \\ -5 & -41 \end{vmatrix} \begin{matrix} -185 \\ -287 \end{matrix} \quad \begin{matrix} -287 + 185 \\ = -102 \end{matrix}$$

$$\begin{vmatrix} 7 & 3 \\ -5 & -7 \end{vmatrix} \begin{matrix} -15 \\ -49 \end{matrix} \quad \begin{matrix} -49 + 15 = \\ -34 \end{matrix}$$

$$\left(\frac{-136}{-34}, \frac{-102}{-34} \right)$$
$$(4, 3)$$