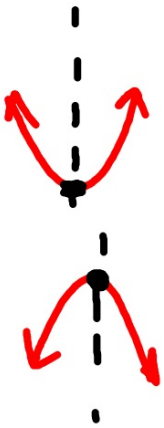
Algebra 2 4.1

Graph quadratic functions

Find and interpret maximum and minimum values of a quadratic function

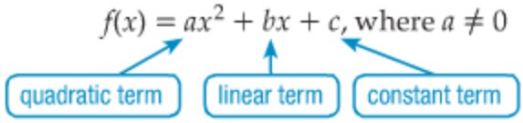
quadratic function
quadratic term
linear term
constant term
parabola
axis of symmetry
y-intercept
vertex
maximum ( )
minimum ( )
axis of symmetry
x=



whiteboards

$f(x) = 0^2 + 0 + 1$

$f(x) = 2x^2$



## KeyConcept Graph of a Quadratic Function—Parabola

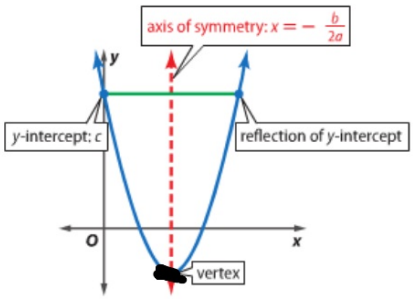
**Words**

Consider the graph of $y = ax^2 + bx + c$, where $a \neq 0$.

- The $y$-intercept is $a(0)^2 + b(0) + c$ or $c$.
- The equation of the axis of symmetry is $x = -\frac{b}{2a}$.
- The $x$-coordinate of the vertex is $-\frac{b}{2a}$.

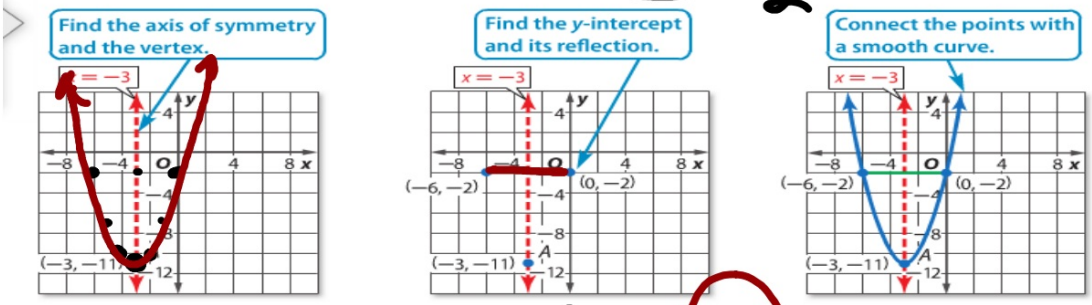
**Model**

D: ARN



p. 220

Now you can use the axis of symmetry to help plot points and graph a parabola. For $y = x^2 + 6x - 2$ below, the axis of symmetry is $x = -\frac{b}{2a} = \frac{-6}{2(1)} = \frac{-6}{2}$ or $x = -3$.




Domain & range?

D: ARN

R: $y \geq -11$

| | $x^2 + 6x - 2$ | |
|---|---|---|
| −3 | 9 + −18 − 2 | −11 |
| −2 | 4 + −12 − 2 | −10 |
| −1 | 1 + −6 − 2 | −7 |
| 0 | 0 + 0 − 2 | −2 |

pos. a

neg a

**Example 2** Axis of Symmetry, *y*-intercept, and Vertex

Consider $f(x) = 1x^2 + 4x - 3$.

$x = \frac{-4}{2} = -2$

a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.

(0, -3)

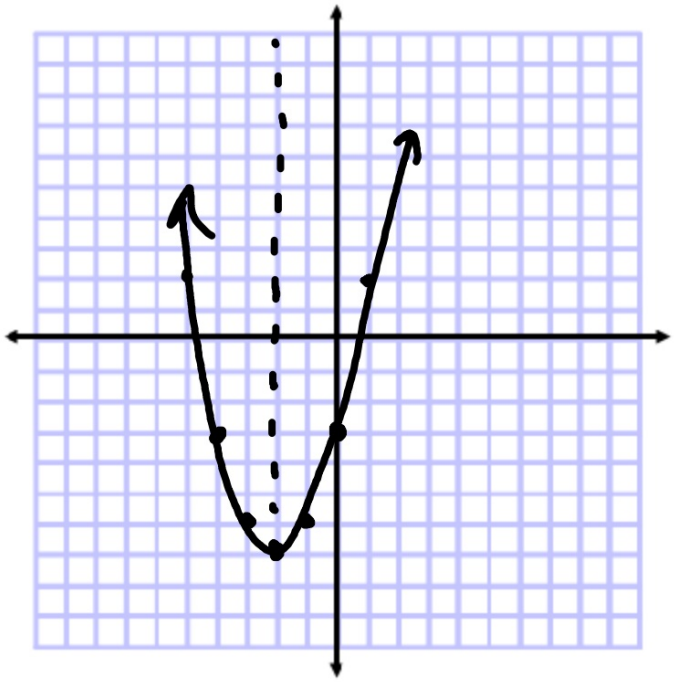
$x = -2$

b. graph the function

c. What is the domain & range?

| | $x^2+4x-3$ | |
|---|---|---|
| -2 | $4 + -8 - 3$ | -7 |
| -1 | $1 + -4 - 3$ | -6 |
| 1 | $1 + 4 - 3$ | 2 |

D ARN  R $y \geq -7$



## KeyConcept Maximum and Minimum Value

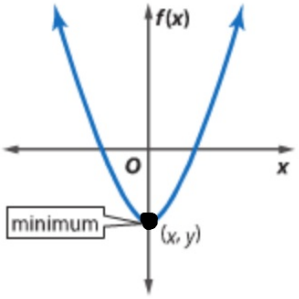
**Words**

The graph of $f(x) = ax^2 + bx + c$, where $a \neq 0$,

- opens up and has a minimum value when $a > 0$, and
- opens down and has a maximum value when $a < 0$.

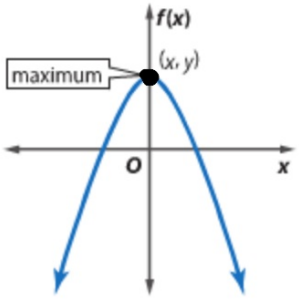
**Model**

$a$ is positive.

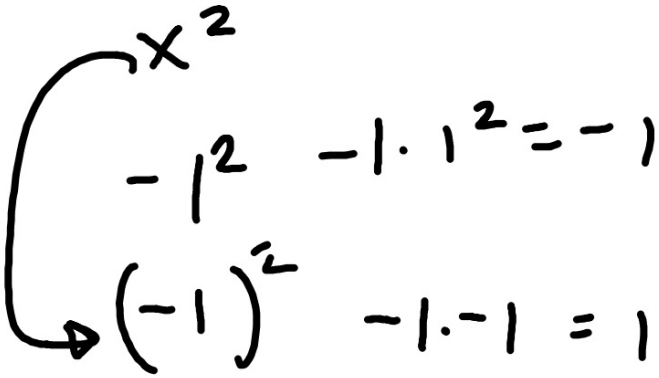



The $y$-coordinate is the minimum value.

$a$ is negative.




The $y$-coordinate is the maximum value.


$x^2$
$-1^2$ $-1 \cdot 1^2 = -1$
$(-1)^2$ $-1 \cdot -1 = 1$

**Example 3** Maximum or Minimum Values

Consider $f(x) = -4x^2 + 12x + 18$.

a. Determine whether the function has a *maximum* or *minimum* value.

$x = \frac{-12}{-8} = 1.5$

b. State the maximum or minimum value of the function.

(1.5, 30)

c. State the domain and range of the function.

D: $x$ ARN

R: $y \leq 30$

| 1.5 | $-6+18+18$ | 30 |
|---|---|---|

**WatchOut!**

**Maxima and Minima** The terms *minimum point* and *minimum value* are not interchangeable. The minimum point on the graph of a quadratic function is the ordered pair that describes the location of the vertex. The minimum value of a function is the *y*-coordinate of the minimum point. It is the smallest value obtained when f(*x*) is evaluated for all values of *x*.

## GuidedPractice

**3.** Consider $f(x) = 4x^2 - 24x + 11$.

**A.** Determine whether the function has a maximum or minimum value.

**B.** State the maximum or minimum value of the function.

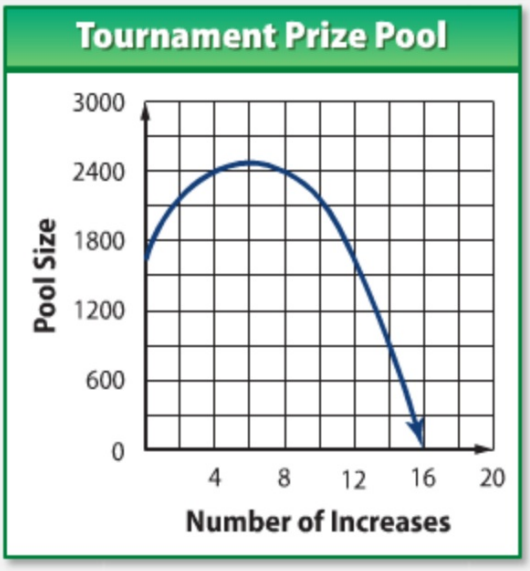
**C.** State the domain and range of the function.

**StudyTip**

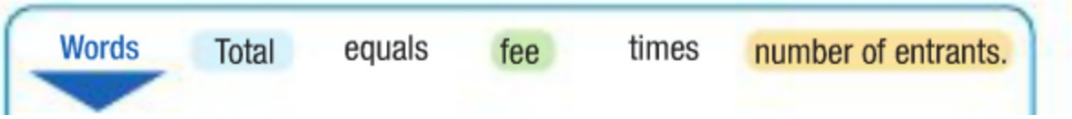
**Domain and Range** The domain of a quadratic function will always be all real numbers. The range will either be all real numbers less than or equal to the maximum or all real numbers greater than or equal to the minimum.

- Eddie is organizing a charity tournament. He plans to charge a $20 entry fee for each of the 80 players. He recently decided to raise the entry fee by $5, and 5 fewer players entered with the increase. He used this information to determine how many fee increases will maximize the money raised.

  The quadratic function at the right represents this situation. The tournament prize pool increases when he first increases the fee, but eventually the pool starts to decrease as the fee gets even higher.



**a. How much should Eddie charge in order to maximize charity income?**



## GuidedPractice

**4.** Suppose a different tournament that Eddie organizes has 120 players and the entry fee is $40. Each time he increases the fee by $5, he loses 10 players. Determine what the entry fee should be to maximize the value of the pool.