Algebra 2 4.6

Solve quadratic equations by using the quadratic formula x^2

Use the discriminant to determine the number and type of roots for a quadratic equation

standard form (of a quadratic) $ax^2 + bx + c = 0$

a,b,c

radical $\sqrt{}$

discriminant

quadratic formula

complex number $a+bi$

conjugate pair $3+5i$ $3-5i$

irrational number

exact answer $\sqrt{26}$ $5.099019514\ldots$

QF song

exact rounded off

$2x^2 + 8x - 6 = 0$

$$\frac{2x^2}{2} + \frac{8x}{2} - \frac{6}{2} = \frac{0}{2}$$

$$x^2 + 4x - 3 = 0$$
$$\quad\quad +3 \quad +3$$
$$x^2 + 4x + 4 = 3 + 4$$
$$\sqrt{(x+2)^2} = \sqrt{7}$$
$$x + 2 = \pm\sqrt{7}$$
$$-2 \quad -2$$
$$x = -2 \pm \sqrt{7}$$

Solve by completing the square
(keep track of the steps used)

1. Divide $a \neq 1$
2. move c
3. $\frac{b}{2}$ or $\frac{1}{2} \cdot b$
4. b^2 to both
5. simplify
6. $\sqrt{\ }$ $\pm\sqrt{\ }$
7. solve for X

Same steps

1 Quadratic Formula

Quadratic Formula You have found solutions of some quadratic equations by graphing, by factoring, and by using the Square Root Property. There is also a formula that can be used to solve any quadratic equation. This formula can be derived by solving the standard form of a quadratic equation.

$$ax^2 + bx + c = 0$$

General Case

$\frac{b}{2a} \cdot \frac{b}{2a}$

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$\frac{1}{2} \cdot \frac{b}{a}$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \left(-\frac{c}{a}\cdot\frac{4a}{4a} + \frac{b^2}{4a^2}\right)$$

$\frac{b}{2a}$

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

KeyConcept Quadratic Formula

Words The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, are given by the following formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example $$\boxed{x^2 + 5x + 6 = 0} \rightarrow x = \frac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$$

Quadratic formula song

QF song!

Must be in standard form... =0

Example 1 Two Rational Roots

Solve $x^2 - 10x = 11$ by using the Quadratic Formula.

$-11 \quad -11$

$1x^2 - 10x - 11 = 0$

a b c

$$x = \frac{10 \pm \sqrt{-10 \cdot -10 - 4 \cdot 1 \cdot -11}}{2 \cdot 1}$$

$$x = \frac{10 \pm \sqrt{100 + 44}}{2}$$

$$x = \frac{10 \pm 12}{2}$$

$x = \frac{22}{2} = 11$

$x = \frac{-2}{2} = -1$

Guided Practice

QF

Solve each equation by using the Quadratic Formula.

1A. $x^2 + 6x = 16$

$x = 2$

$x = -8$

1B. $2x^2 + 25x + 33 = 0$

$x^2 + 6x - 16 = 0$

$$x = \frac{-6 \pm \sqrt{36 - 4 \cdot 1 \cdot -16}}{2}$$

$$= \frac{-6 \pm \sqrt{100}}{2} = \frac{-6 \pm 10}{2}$$

$$x = \frac{-8 \pm \sqrt{0}}{2} = -4$$

Example 2 One Rational Root

Solve $x^2 + 8x + 16 = 0$ by using the Quadratic Formula.

$$x = \frac{-3 \pm \sqrt{27}}{2}$$

$$\frac{-3 \pm 3\sqrt{3}}{2}$$

27

3 9

3 3

Solve each equation by using the Quadratic Formula.

2A. $x^2 - 16x + 64 = 0$

2B. $x^2 + 34x + 289 = 0$

Example 3 Irrational Roots

Solve $2x^2 + 6x - 7 = 0$ by using the Quadratic Formula.

a. $d = ?$ $6 \cdot 6 - 4 \cdot 2 \cdot -7$ $d = 92$ 2 real, irr

$36 + 56$

b. solve

$$\frac{-6 \pm \sqrt{92}}{4} = \frac{-6 \pm 2\sqrt{23}}{4}$$

92

2 46

2 23

GuidedPractice

Solve each equation by using the Quadratic Formula.

3A. $3x^2 + 5x + 1 = 0$

3B. $x^2 - 8x + 9 = 0$

$B^2 - 4AC$

Example 4 Complex Roots

~~Solve~~ $x^2 - 6x = -10$ ~~by using the Quadratic Formula.~~

$+10 \quad +10$

$d = -6 \cdot -6 - 4 \cdot 1 \cdot 10$

$= 36 - 40$

$= -4$

$x = \frac{6 \pm \sqrt{-4}}{2}$

$x = \frac{6 \pm 2i}{2} = 3 \pm i$

$d = 100$ $d = 90$

Guided Practice

~~Solve each equation by using the Quadratic Formula.~~

4A. $3x^2 + 5x + 4 = 0$

a b c

$d = 25 - 4 \cdot 3 \cdot 4$

$= 25 - 48$

$= -23$

4B. $x^2 - 4x = -13$

$x^2 - 4x + 13 = 0$

$d = 16 - 4 \cdot 1 \cdot 13$

$= -36$

2 Roots and the Discriminant In the previous examples, observe the relationship between the value of the expression under the radical and the roots of the quadratic equation. The expression $b^2 - 4ac$ is called the **discriminant**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \leftarrow \textbf{discriminant}$$

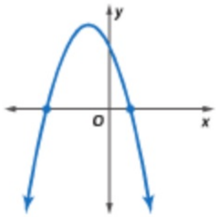
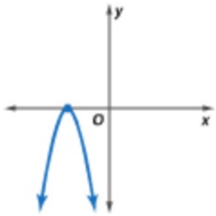
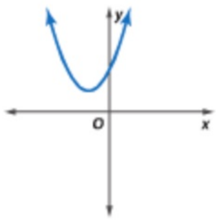
(What is UNDER the radical?
It's a COMPONENT of the QF))

$b^2 - 4ac = d$

nature of roots...

KeyConcept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
pos. $b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
pos $b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
zero $b^2 - 4ac = 0$	1 real rational root (D.R.)	
neg $b^2 - 4ac < 0$	2 complex roots	

√

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(double root)