Algebra 2 4.4

Perform operations with imaginary numbers*

Perform operations with complex numbers*

radical

simplify (by casting out pairs) geometry

square root property

real number

imaginary unit

pure imaginary numbers

complex numbers

complex conjugate

*New concept--first time you have seen this idea!

Triangle puzzle (if time)

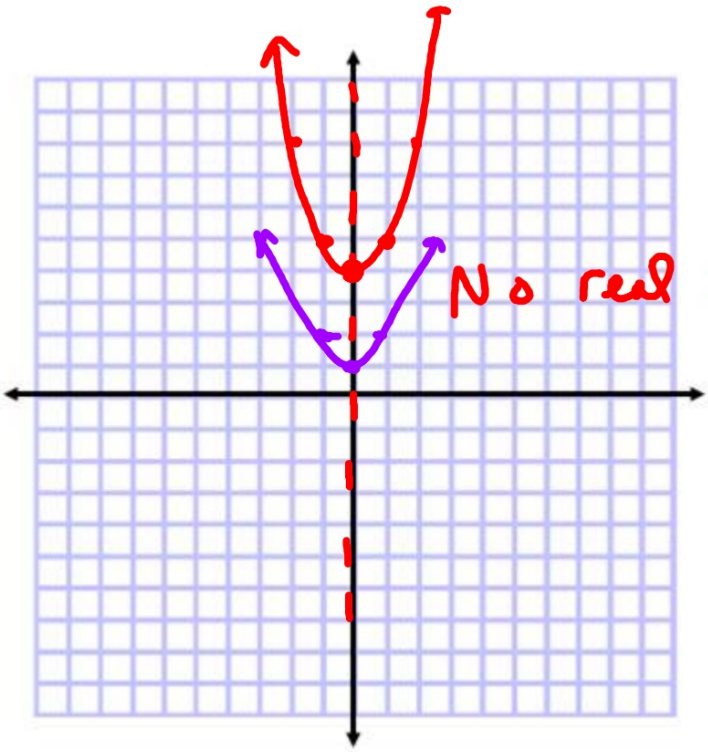
No solution = no real solution!

Solve by graphing
$x^2 + 4 = 0$

$y = x^2 + 4$

$x = -\frac{b}{2a} = \frac{0}{2} = 0$

x	x^2+4	
0	0^2+4	4
1	$1+4$	5
2	$4+4$	8



No real solution

$x^2 + 1 = 0$

$-1 \quad -1$

$\sqrt{x^2} = \sqrt{-1}$

Graph the related function

✗ Solve using algebra

$x = \sqrt{-1}$

$(\quad)^2 = -1$

non real

domain error

$i =$

The **imaginary unit i** is defined to be

$$\boxed{i = \sqrt{-1}.}$$

$6\sqrt{-1}$

$-2i = -2\sqrt{-1}$

$\sqrt{3} \cdot \sqrt{-1}$

Numbers of the form $6i$, $-2i$, and $i\sqrt{3}$ are called **pure imaginary numbers**. Pure imaginary numbers are square roots of negative real numbers. For any positive real number b, $\sqrt{-b^2} = \sqrt{b^2} \cdot \sqrt{-1}$ or bi.

Guided Practice

1A. $\sqrt{-18}$

$18 = 9 \cdot 2$, $9 = 3 \cdot 3$

$\sqrt{18 \cdot -1}$

$\sqrt{18}\ \sqrt{-1}$

$3\sqrt{2} \cdot \sqrt{-1}$

* $3\sqrt{2}\, i = 3i\sqrt{2}$ *

$3\sqrt{-2}$ ~~-1~~

1B. $\sqrt{-125}$

$125 = 25 \cdot 5$, $25 = 5 \cdot 5$

$\sqrt{125 \cdot -1}$

$5i\sqrt{5} = 5\sqrt{5}\, i$

"Casting out pairs" to simplify the real part
What about the negative?

$$i^1 = \sqrt{-1}$$

$$i^2 = i\,i = \sqrt{-1} \cdot \sqrt{-1} = -1$$

$$i^3 = i\,i\,i = \underbrace{\sqrt{-1} \cdot \sqrt{-1}} \cdot \sqrt{-1} = -1\,i = -i$$

$$i^4 = i\,i\,i\,i = \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} \cdot \underbrace{\sqrt{-1} \cdot \sqrt{-1}}_{-1} = 1$$

$$i^{25}$$

$$i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \; i$$

$$1 \cdot 1 \cdot 1 \cdot 1 \cdot 1\, i$$

$$i$$

Example 2 Products of Pure Imaginary Numbers

Simplify.

a. $-5i \cdot 3i = -5 \cdot 3 \cdot i \cdot i$

$= -15 \cdot -1$

$= 15$

$i = \sqrt{-1}$

$ii =$

GuidedPractice

2A. $3i \cdot 4i$

$3 \cdot 4 \cdot i \cdot i$

$12(-1)$

-12

2B. $\sqrt{-20} \cdot \sqrt{-12}$

240

24 10

6 4 $\cancel{2}$ 5

3 $\cancel{2}$ $\cancel{2}$ $\cancel{2}$

$\sqrt{240 \cdot -1 \cdot -1}$

$2 \cdot 2\sqrt{3 \cdot 5}\ \underbrace{i\, i}_{-1}$

$-4\sqrt{15}$

2C. i^{31}

$i^{28}\ i^{3}$

$1\ \underbrace{i\, i}_{-1}\, i$

$-1\, i$

$-i$

Example 3 Equation with Pure Imaginary Solutions

Solve $x^2 + 64 = 0$.

$-64 \quad -64$

$\sqrt{x^2} = \sqrt{-64}$

$x = \sqrt{64}\sqrt{-1}$

$= \pm 8i$

$(\ )^2 = 25$

5 or -5

Guided Practice

Solve each equation.

3A. $4x^2 + 100 = 0$

3B. $x^2 + 4 = 0$

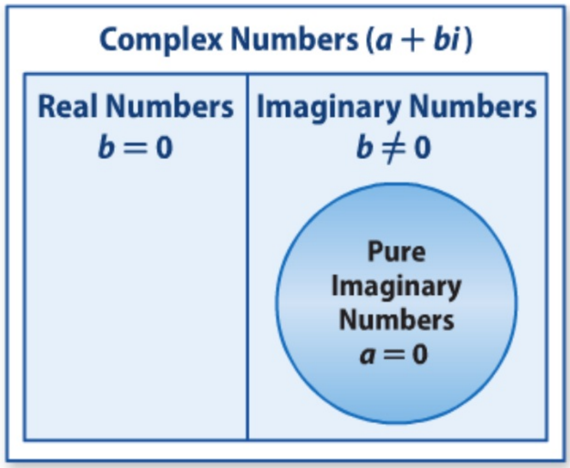
2 Operations with Complex Numbers Consider $2 + 3i$. Since 2 is a real number and $3i$ is a pure imaginary number, the terms are not like terms and cannot be combined. This type of expression is called a **complex number**.

KeyConcept Complex Numbers

Words	A complex number is any number that can be written in the form $a + bi$, where a and b are real numbers and i is the imaginary unit. a is called the real part, and b is called the imaginary part.
Examples	$5 + 2i$ $1 - 3i = 1 + (-3)i$

The Venn diagram shows the set of complex numbers.

- If $b = 0$, the complex number is a real number.
- If $b \neq 0$, the complex number is imaginary.
- If $a = 0$, the complex number is a pure imaginary number.



Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal. That is, $a + b\boldsymbol{i} = c + d\boldsymbol{i}$ if and only if $a = c$ and $b = d$.

real = real   imag = imag

Example 4 Equate Complex Numbers

Find the values of x and y that make $3x - 5 + (y - 3)i = 7 + 6i$ true.