Algebra 2 7.5

Simplify and evaluate expressions using the properties of logarithms

Solve logarithmic equations

logarithm = exponent

product rule $x^2 y^2 \cdot x^4$

quotient rule $\frac{a^3}{a^2}$

power^power $(x^1 y^2)^3 = x^3 y^6$

$10^2 = 100$

$\log_{10} 100 = 2$

log=exponent so same rules apply

$x^2 x^3$

KeyConcept Product Property of Logarithms

Words	The logarithm of a product is the sum of the logarithms of its factors.
Symbols	For all positive numbers a, b, and x, where $x \neq 1$, $\log_x ab = \log_x a + \log_x b$.
Example	$\log_2 [(5)(6)] = \log_2 5 + \log_2 6$

$\log_2(30) = \log_2(5 \cdot 6) = \log_2 5 + \log_2 6$

Example 3 Power Property of Logarithms

Given $\log_2 5 \approx 2.3219$, approximate the value of $\log_2 25$.

BOGO: $\log_2 2 = 1$

Write as combination of 2s & 5s
Check answer

$\log_2(5 \cdot 5) = \log_2 5 + \log_2 5$

$= 2.3219 + 2.3219$

$= 4.6438$

4.6438

2

$\log_2 50$

$\log_2(2 \cdot 5 \cdot 5)$

$1 + 2.3219 + 2.3219$

$= 5.6438$

$2(\ ) =$

192
4 · 48
4 · 12
3 · 4

27
81
12

BOGO: $\log_4 4 = 1$

Example 1 Use the Product Property

Use $\log_4 3 \approx 0.7925$ to approximate the value of $\log_4 192$.

$\log_4(192) = \log_4(3 \cdot 4 \cdot 4 \cdot 4)$

$= 0.7925 + 1 + 1 + 1$

$= 3.7925$

$4(\ ) =$

How can I write 192?
write using factors of :
3 (given) and 4 (base)
combine by adding exponents
check answer

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1. Use $\log_4 2 = 0.5$ to approximate the value of $\log_4 32$.

BOGO: $\log_4 4 = 1$

write using factors of 2 and 4 (base)

$\log_4(32) = \log_4(4 \cdot 4 \cdot 2)$

$= 1 + 1 + .5$

$= 2.5$

GuidedPractice

3. Given $\log_3 7 \approx 1.7712$, approximate the value of $\log_3 49$.

BOGO: $\log_3 3$

Write as combination of 3s & 7s (base)

$\log_3(7 \cdot 7) = 1.7712 + 1.7712$

$= 3.5424$

Log=exp so follow same rules x^5/x^2

$$\frac{x^5}{x^2}$$

KeyConcept Quotient Property of Logarithms

Words	The logarithm of a quotient is the difference of the logarithms of the numerator and the denominator.
Symbols	For all positive numbers a, b, and x, where $x \neq 1$, $\log_x \frac{a}{b} = \log_x a - \log_x b$.
Example	$\log_2 \frac{5}{6} = \log_2 5 - \log_2 6$

acid 0 — 7 — 14 base

Hint: divide out -1 to solve for log
Write in exp form

Real-World Example 2 Quotient Property

SCIENCE The pH of a substance is defined as the concentration of hydrogen ions $[H^+]$ in moles. It is given by the formula $pH=-\log_{10}H^+$. Find the amount of hydrogen in a liter of acid rain that has a pH of 4.2.

H^+

Note change in formula

$$pH=-\log_{10}H^+$$

$$\frac{pH}{-1}=\frac{-1\cdot\log_{10}H^+}{-1}$$

$$-pH=\log_{10}H^+$$

$$-4.2=\log_{10}H^+$$

Write in exp form

$$10^{(-4.2)}=H^+$$

0.000063096

$\approx 6.3\times10^{-5}$

log=exp so follow same rules: $(x^2)^4 = x^8$

KeyConcept Power Property of Logarithms

Words	The logarithm of a power is the product of the logarithm and the exponent.
Symbols	For any real number p, and positive numbers m and b, where $b \neq 1$, $\log_b m^p = p \log_b m$.
Example	$\log_2 6^5 = 5 \log_2 6$

hint: exponents mean repeated multiplication

What does it mean if exponents are added?

Example 4 Solve Equations Using Properties of Logarithms

Solve $\log_6 x + \log_6 (x - 9) = 2$.

$x = 12$

$\log_6 x(x-9) = 2$

$\log_6 (x^2 - 9x) = 2$

$6^2 = x^2 - 9x$

$0 = x^2 - 9x - 36$

-36, -12, 3, -9

$0 = (x-12)(x+3)$

$x - 12 = 0$

$x = 12$

$x + 3 = 0$

$x = -3$

4B. $\log_6 x + \log_6 (x + 5) = 2$

$\log_6 x(x+5) = 2$

$\log_6 x^2 + 5x = 2$

$36 = x^2 + 5x$

$0 = x^2 + 5x - 36$

$(x+9)(x-4)$

-36

$9 \quad -4$

5

$x + 9 = 0$

$x = -9$

$x - 4 = 0$

$x = 4$

What does it mean if exponents are multiplied?

$(x^{-3})^2 =$

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4A. $2 \log_7 x = \log_7 27 + \log_7 3$

$\log_7 (x)^2 = \log_7 (27 \cdot 3)$

$\log_7 x^2 = \log_7 81 \qquad x = 9$

$x^2 = 81$

$x = \pm 9$

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2. $\log_{10} x = \log_{10} x + \log_{10} 4$

$\log_{10} x^2 = \log_{10} 4x$

~~$x \neq 0$~~ $x = 4$

$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$