

Algebra 2            5.3

- Evaluate polynomial functions
- Identify general shapes of polynomial function graphs

polynomial    *number, variable, products*  
function      *terms + or -*

Quiz 5.1-5.2 Wed.

\* parent graph  
degree       *$f(x) = 1x^2 + 6x - 3$*   
coefficient    *y =*  
leading coefficient  
function notation  
end behavior  
even function  
odd function

activity: Kroon says

$$2x^2 + 3x + 1$$

$$x - 4$$

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$$-8x^2 - 12x - 4$$

$$2x^3 + 3x^2 + x$$

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$$2x^3 - 5x^2 - 11x - 4$$

Polynomial	Expression	Degree	Leading Coefficient	
→ Constant	$12 \cdot x^0 = 12 - 1 = 12$	0	12	*technically
→ Linear	$4x - 9$	1	4	
→ Quadratic	$5x^2 - 6x - 9$ $-6x - 9 + 5x^2$	2	5	
→ Cubic	$8x^3 + 12x^2 - 3x + 1$	3	8	
→ General	$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$	$n$	$a_n$	

↑



### Example 1 Degrees and Leading Coefficients

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

a.  $8x^5 - 4x^3 + 2x^2 - x - 3$

~~b.  $12x^2 - 3xy + 6x$~~

c.  $3x^4 + 6x^3 + 4x^8 + 2x$

**Guided**Practice

**1A.**  ~~$5x^3 - 4x^2 - 8x + \frac{4}{x}$~~

**1B.**  $5x^6 - 3x^4 + 12x^3 - 14$

**1C.**  $8x^4 - 2x^3 - x^6 + 3$

# GEMA

$$f(x) = 3x + 5$$

$$f(2) = 3 \cdot 2 + 5 = 11$$

$$f(-5) = 3 \cdot (-5) + 5 = -10$$

$$f(x) = 3x^2 - 5$$

$$f(4) = 3 \cdot 4^2 - 5 = 43$$

$$f(-3) = 3 \cdot x \cdot x - 5$$

$$= 3 \cdot (-3) \cdot (-3) - 5$$

$$= 27 - 5 = 22$$

Algebra 1

$$-3^2 \quad (-3)^2$$
$$-1 \cdot 3^2$$

$$f(x) = x^2 + 2x - 3$$

$$1. f(2a) - f(a)$$

$$2. f(3c) - 4f(c)$$

What is the function?

Have a plan... order of ops.

$$= 3a^2 + 2a$$

$$f(2a) - f(a) = \begin{pmatrix} x \cdot x + 2x - 3 \\ 2a \cdot 2a + 2 \cdot 2a - 3 \\ 4a^2 + 4a - 3 \end{pmatrix} - \begin{pmatrix} x \cdot x + 2x - 3 \\ a \cdot a + 2 \cdot a - 3 \\ a^2 + 2a - 3 \end{pmatrix} = 3a^2 + 2a$$

### Example 3 Function Values of Variables

3. Find  $f(3c - 4) - 5f(c)$  if  $f(x) = x^2 + 2x - 3$ .

$$\begin{array}{r} 3c - 4 \\ 3c - 4 \\ \hline a^2 - 12c + 16 \\ -12c \\ \hline 9c^2 - 24c + 16 \end{array}$$

$$f(3c-4) - 5f(c) = \begin{pmatrix} x \cdot x + 2x - 3 \\ (3c-4)(3c-4) + 2(3c-4) - 3 \\ 9c^2 - 24c + 16 + 6c - 8 - 3 \end{pmatrix} + (-1) \begin{pmatrix} 5 \cdot (c^2 + 2c - 3) \\ 5c^2 + 10c - 15 \end{pmatrix} = 4c^2 - 28c + 20$$

Guided Practice

3A. Find  $g(5a - 2) + 3g(2a)$  if  $g(x) = x^2 - 5x + 8$ .

$$\begin{aligned} & \overbrace{(5a-2)}^{x^2} \overbrace{(5a-2)}^{x^2} - 5(5a-2) + 8 + \left( 3(2a \cdot 2a - 5 \cdot 2a + 8) \right) \\ & (25a^2 - 20a + 4 - 25a + 10 + 8) + (12a^2 - 30a + 24) \\ & \overline{25a^2 - 45a + 22} + 12a^2 - 30a + 24 \end{aligned}$$

~~order of operations~~  
order of operations...

$$37a^2 - 75a + 46$$



**3B.** Find  $h(-4d + 3) - 0.5h(d)$  if  $h(x) = 2x^2 + 5x + 3$ .

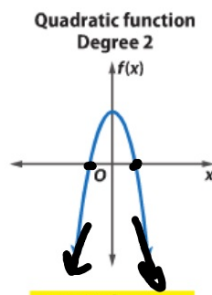
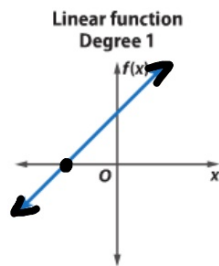
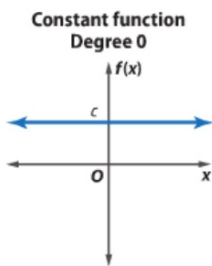
$h(-4d+3)$

$h(d)$

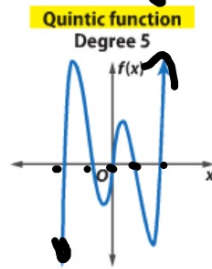
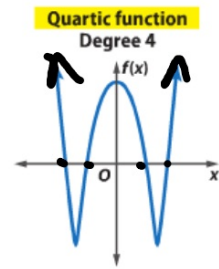
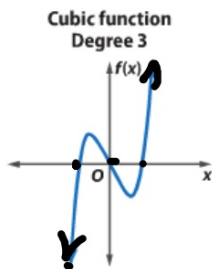
order of operations

**2 Graphs of Polynomial Functions** The general shapes of the graphs of several polynomial functions show the *maximum* number of times the graph of each function may intersect the  $x$ -axis. This is the same number as the degree of the polynomial.

p 324



degree =  
degree - 1 =



$d - 1 = \text{t.p.}$   
(at most)

$d = \text{x-int}$   
(at most)

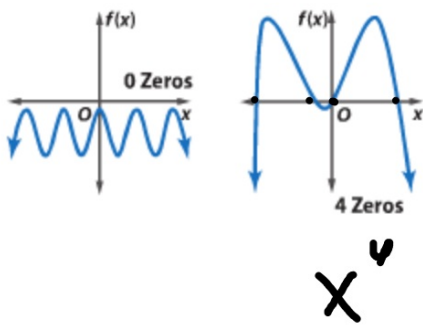
Kroon says...

**Key Concept** Zeros of Even- and Odd-Degree Functions

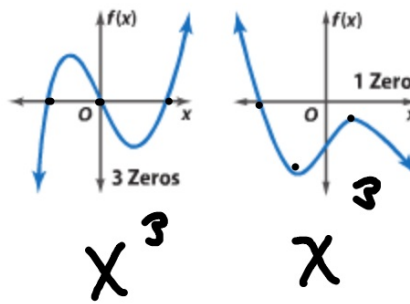
Odd-degree functions will always have an odd number of real zeros. Even-degree functions will always have an even number of real zeros or no real zeros\* at all.

\*zero is an even number

Even-Degree Polynomials



Odd-Degree Polynomials



**KeyConcept** End Behavior of a Polynomial Function



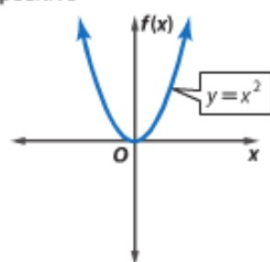
**Degree:** even

**Leading Coefficient:** positive

**End Behavior:**

$f(x) \rightarrow +\infty$   
as  $x \rightarrow -\infty$

$f(x) \rightarrow +\infty$   
as  $x \rightarrow +\infty$



Domain: all real numbers  
Range: all real numbers  $\geq$  minimum

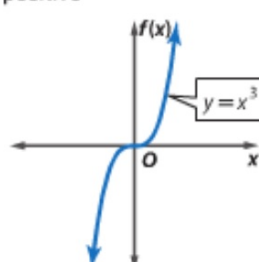
**Degree:** odd

**Leading Coefficient:** positive

**End Behavior:**

$f(x) \rightarrow -\infty$   
as  $x \rightarrow -\infty$

$f(x) \rightarrow +\infty$   
as  $x \rightarrow +\infty$



Domain: all real numbers  
Range: all real numbers

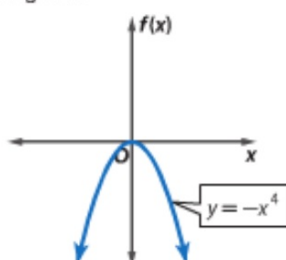
**Degree:** even

**Leading Coefficient:** negative

**End Behavior:**

$f(x) \rightarrow -\infty$   
as  $x \rightarrow -\infty$

$f(x) \rightarrow -\infty$   
as  $x \rightarrow +\infty$



Domain: all real numbers  
Range: all real numbers  $\leq$  maximum

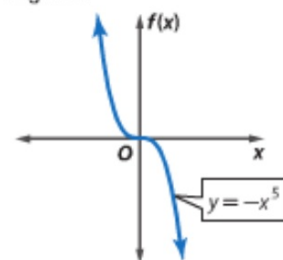
**Degree:** odd

**Leading Coefficient:** negative

**End Behavior:**

$f(x) \rightarrow +\infty$   
as  $x \rightarrow -\infty$

$f(x) \rightarrow -\infty$   
as  $x \rightarrow +\infty$



Domain: all real numbers  
Range: all real numbers

Looks pretty intimidating:  
You just have to know the cod

end behavior: words, code

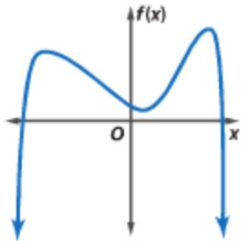


#### Example 4 Graphs of Polynomial Functions

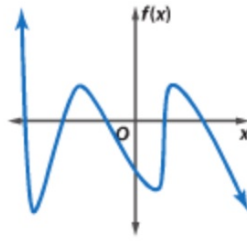
For each graph,

- describe the end behavior,
- determine whether it represents an odd-degree or an even-degree polynomial function, and
- state the number of real zeros.

a.

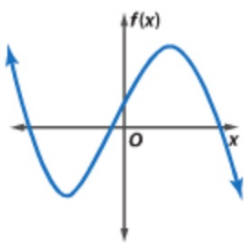


b.



► **Guided** Practice

4A.



4B.

