Algebra 2 5.7

Determine the number and type of roots for a polynomial equation *Ch. 4

Find the zeros of a polynomial function

Write equations from given roots

Fundamental theorem of algebra (how many roots?)

Descarte's rule of signs (type of roots)

complex number*

conjugate pair*

coefficient

integer

Quiz 5.5-5.6

What are steps to solve a quadratic?

$x^2 - 4x + 3 = 0$

3

-3 -1

-4

$(x-3)(x-1) = 0$

$x - 3 = 0$ $x - 1 = 0$

$+3 \quad +3$ $+1 \quad +1$

$x = 3$ $x = 1$

$x - 2$

$x - 5$

$-5x + 10$

$x^2 \quad -2x$

$x^2 - 7x + 10 = 0$

$(x-2)(x-5) = 0$

$x - 2 = 0$ $x - 5 = 0$

$x = 2$ $x = 5$

$-2 \quad -2$ $-5 \quad -5$

Write equation with x=3 and x=-5 as roots (solutions)

$$x^2+2x-15=0$$

$$(x-3)(x+5)=0$$

$$x-3=0 \qquad x+5=0$$

$$\begin{array}{cc} x=3 & x=-5 \\ -3 \quad -3 & +5 \quad +5 \end{array}$$

$$\begin{array}{r} x-3 \\ x+5 \\ \hline 5x \quad -15 \\ x^2 \; -3x \qquad \end{array}$$

$x = 3i \qquad x = -3i$

KeyConcept Complex Conjugates Theorem

Words Let a and b be real numbers, and $b \neq 0$. If $a + b\boldsymbol{i}$ is a zero of a polynomial function with real coefficients, then $a - b\boldsymbol{i}$ is also a zero of the function.

Example If $3 + 4\boldsymbol{i}$ is a zero of $f(x) = x^3 - 4x^2 + 13x + 50$, then $3 - 4\boldsymbol{i}$ is also a zero of the function.

$x = 2+5i \qquad x = 2-5i$

$x^2 + 9 =$

$(x-3i)(x+3i) = 0$

$x-3i = 0 \qquad x+3i = 0$

$x = 3i \qquad x = -3i$

$-3i \quad -3i \qquad +3i \quad +3i$

$x - 3i$

$x + 3i$

$x^2 \quad 3ix \quad -9i^2$

$-3ix \quad -9 \cdot -1$

$$x^2 - 7x + 10 = 0$$

$$(x-5)(x-2) = 0$$

$$(x-5) = 0 \qquad (x-2) = 0$$

$$x = 5 \qquad x = 2$$

Write equations given roots...

Example 4 Use Zeros to Write a Polynomial Function

Write a polynomial function of least degree with integral coefficients, the zeros of which include −1 and 5 − *i*.

These are the solutions

(are they all there?)

What are the factors?

How many factors?

Equations must = something

What is an integer?

$(x+1)(x^2-10x+26)=0$

$x+1=0 \qquad x-5+i=0 \qquad x-5-i=0$

$x=-1 \qquad x=5-i \qquad x=5+i$

$x^3-9x^2+16x+26=0$

$x^2-10x+26$

$x+1$

$x^3 \quad x^2 \quad -10x \quad 26$

$-10x^2 \quad 26x$

$x-5+i$

$x-5-i$

$-ix \quad 5i \quad -i^2$

$-5i$

$-5x+25$

$x^2-5x+ix$

$x^2-10x+26$

+3, −3, −9, −3·3

1. answer
2. make factor
3. ewe
4. eq = 0

GuidedPractice

4. Write a polynomial function of least degree with integral coefficients having zeros that include -1 and $1 + 2i$.

These are the solutions.
What are the factors?
How many factors?
Equations must = something
What is an integer?

$(x+1)(x^2-2x+5)=0$

$(x+1)\underbrace{(x-1-2i)(x-1+2i)}=0$

$x+1=0 \qquad x-1-2i=0 \qquad x-1+2i=0$

$x=-1 \qquad x=1+2i \qquad x=1-2i$

$$\begin{array}{r} x-1-2i \\ x-1+2i \\ \hline 2ix \quad -2i \quad -4i^2 \\ x^2 \quad -x \quad +1 \quad +2i \quad -4(-1) \\ -x \quad -2ix \\ \hline x^2-2x+5 \end{array}$$

$$\begin{array}{r} x^2-2x+5 \\ x+1 \\ \hline x^3 \quad x^2 \quad -2x \quad 5 \\ -2x^2 \quad +5x \end{array}$$

$x^3-x^2+3x+5=0$

GuidedPractice

2. State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $h(x) = -2x^5 + x^4 - 3x^3 - 4x^2 + x + 9$.

(+) 2, 0

(−) 3, 1

(i) 0, 2, 4

WS Skills odd

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros

Review Vocabulary

complex conjugates two complex numbers of the form $a + bi$ and $a - bi$

In Chapter 4, you learned that the product of complex conjugates is always a real number and that complex roots always come in conjugate pairs. For example, if one root of $x^2 - 8x + 52 = 0$ is $4 + 6i$, then the other root is $4 - 6i$.

This applies to the zeros of polynomial functions as well. For any polynomial function with real coefficients, if an imaginary number is a zero of that function, its conjugate is also a zero. This is called the **Complex Conjugates Theorem**.