Algebra 2 5.7

Determine the number and type of roots for a polynomial equation

Find the zeros of a polynomial function *Ch. 4

degree (of an equation) x = at most

zero

factor same $(x-2)(x+3)=0$ Quiz 5.5-5.6 Fri.

root

$x-2=0$ $x+3=0$

$x=2$ $x=-3$

x-intercept

→ Fundamental theorem of algebra

→ Descarte's rule of signs

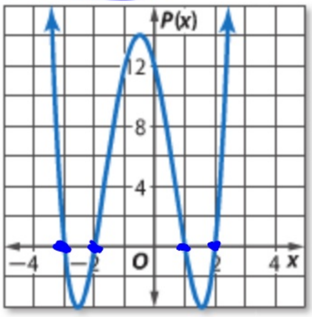
complex number*

conjugate pair*

ConceptSummary Zeros, Factors, Roots, and Intercepts

Consider the polynomial function $\boxed{P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12}$.

The zeros of $P(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$ are -3, -2, 1, and 2.



What do you notice?

KeyConcept Corollary to the Fundamental Theorem of Algebra

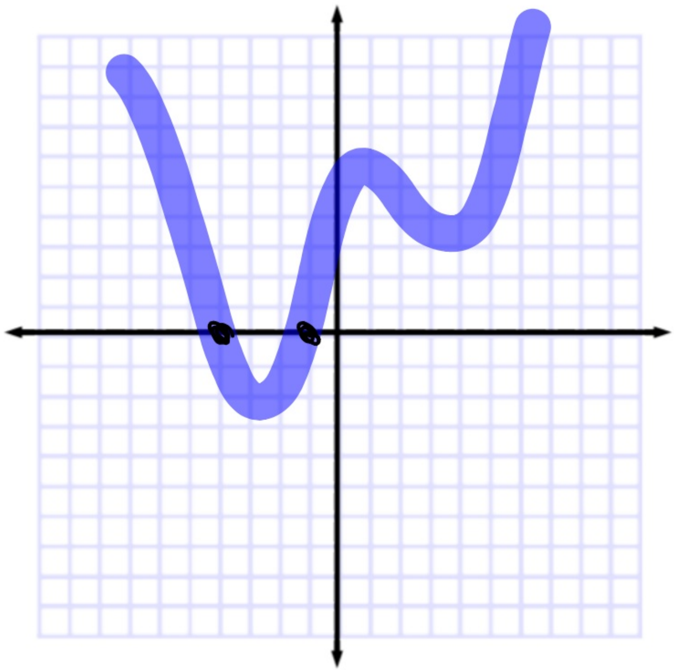
Words A polynomial equation of degree n has exactly n roots in the set of complex numbers, including repeated roots.

Example

$x^3 + 2x^2 + 6$	$4x^4 - 3x^3 + 5x - 6$	$-2x^5 - 3x^2 + 8$
3 roots	4 roots	5 roots

Similarly, an nth degree polynomial function has exactly n zeros.

What would be happening on the graph if y changes from + to -?
So if real roots (zeros) cause sign changes...

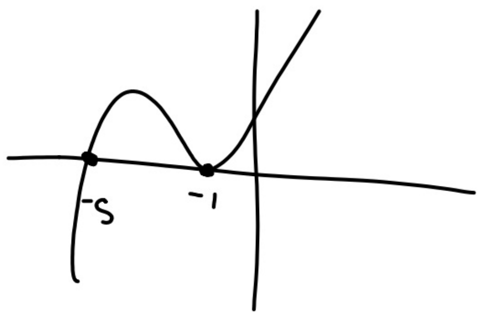


Twists and turns (max/min) on graph are caused by the presence of roots:

- Real roots cause x-intercepts
- Imaginary roots cause max/min (mountains/valleys) but not x-intercept
- Total number of roots = degree

could be a double root

pairs



KeyConcept Descartes' Rule of Signs

Let $P(x) = a_n x^n + \cdots + a_1 x + a_0$ be a polynomial function with real coefficients. Then

- the number of positive real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an even number, and
- the number of negative real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$, or is less than this by an even number.

positive: use f(x)
negative: use f(-x)
What is the deal with "less by an even number" ?

trust me for now... will talk later

Example 2 Find Numbers of Positive and Negative Zeros

State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $f(x) = x^6 + 3x^5 - 4x^4 - 6x^3 + x^2 - 8x + 5$.

$-6(-x)^3 + (-x)^2 - 8(-x)$

f(x)

$f(-x) = (-x)^6 + 3(-x)^5 - 4(-x)^4$

$f(-x) = x^6 - 3x^5 - 4x^4 + 6x^3 + x^2 + 8x + 5$

+ 4, 2, 0

− 2, 0

imag 0, 2, 4, 6

Number of possible roots decreases by even numbers...
Why? creates a squiggle but not a crossing point
What is the least number of real zeros?

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros

$h(-x) = 2(-x)^5 + (-x)^4 + 3(-x)^3 - 4(-x)^2 - -x$

GuidedPractice

$= -2x^5 + x^4 - 3x^3 - 4x^2 + x + 9$

2. State the possible number of positive real zeros, negative real zeros, and imaginary zeros of $h(x) = 2x^5 + x^4 + 3x^3 - 4x^2 - x + 9$.

(+) 2, 0

(−) 3, 1

imag 0, 2, 4

Number of Positive Real Zeros	Number of Negative Real Zeros	Number of Imaginary Zeros	Total Number of Zeros

1	80
~~2~~	40
~~4~~	20
~~5~~	16
10	8

(Roots...) $x^4-18x^2-12x+80$

Example 3 Use Synthetic Substitution to Find Zeros

Find all of the zeros of $f(x) = x^4 - 18x^2 + 12x + 80$.

Try the factors of c... trust me :)

$\oplus$ 2, 0

$\ominus$ 2, 0

imag 0, 2, 4

$$\begin{array}{r|rrrrr} -4 & 1 & 0 & -18 & 12 & 80 \\ & \downarrow & -4 & 16 & 8 & -80 \\ \hline & \boxed{1 \quad -4 \quad -2 \quad 20} & & & & 0 \end{array}$$

$$\begin{array}{r|rrrr} -2 & 1 & -4 & -2 & 20 \\ & \downarrow & -2 & 12 & -20 \\ \hline & 1 & -6 & 10 & 0 \end{array}$$

$x = -4$

$x = -2$

$x = 3+i$

$x = 3-i$

10, 6

$x^2-6x+10$

$$x = \frac{6 \pm \sqrt{36-40}}{2} = \frac{6 \pm \sqrt{-4}}{2} = \frac{6}{2} \pm \frac{2i}{2}$$

Guided Practice

3. Find all of the zeros of $h(x) = x^3 + 2x^2 + 9x + 18$.

1 18

2 9

3 6

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 9 & 18 \\ & \downarrow & -2 & 0 & -18 \\ \hline & 1 & 0 & 9 & 0 \end{array}$$

$x^2 + 9 = 0$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm$

$x = 0 \pm \sqrt{}$

$x = -2$

$x = 3i$

$x = -3i$

imag>>squiggles... always cong pairs

Review Vocabulary

complex conjugates two complex numbers of the form $a + bi$ and $a - bi$

In Chapter 4, you learned that the product of complex conjugates is always a real number and that complex roots always come in conjugate pairs. For example, if one root of $x^2 - 8x + 52 = 0$ is $4 + 6i$, then the other root is $4 - 6i$.

This applies to the zeros of polynomial functions as well. For any polynomial function with real coefficients, if an imaginary number is a zero of that function, its conjugate is also a zero. This is called the **Complex Conjugates Theorem**.

This is why the number of roots decreases by two every time...

KeyConcept Complex Conjugates Theorem

Words	Let a and b be real numbers, and $b \neq 0$. If $a + bi$ is a zero of a polynomial function with real coefficients, then $a - bi$ is also a zero of the function.
Example	If $3 + 4i$ is a zero of $f(x) = x^3 - 4x^2 + 13x + 50$, then $3 - 4i$ is also a zero of the function.

Write an equation with solutions of x= 3 and x= -2

17-35

Write an equation with a solution of x= 2i

reminder:
equations have to = something...

Write an equation with a solution of $x = 3 + 2i$

Example 4 Use Zeros to Write a Polynomial Function

PT

Write a polynomial function of least degree with integral coefficients, the zeros of which include -1 and $5 - i$.

GuidedPractice

4. Write a polynomial function of least degree with integral coefficients having zeros that include -1 and $1 + 2i$.