Algebra 2 5.6

Evaluate functions using synthetic substitution
Determine whether a binomial is a factor of a given polynomial

X remainder theorem R = 0

5 45

X synthetic substitution if remain = 0

depressed polynomial

factor theorem

$f(a) = 4a^2 - 3a + 6$ by $a - 2$.

$a - 2 = 0$
$a = 2$

Method 1 Long Division

Method 2 Synthetic Division

$a - 2 \overline{)\, 4a^2 - 3a + 6}$

$$\begin{array}{r|rrr} 2 & 4 & -3 & 6 \\ & \downarrow & 8 & 10 \\ \hline & 4 & 5 & 16 \end{array}$$

$f(2) = 4(2^2) - 3(2) + 6$
$16 - 6 + 6 = 16$

remainder= $f(2)$.

$2(6)^4 - 5(6)^2 + 8(6) - 7$

Example 1 Synthetic Substitution

If $f(x) = 2x^4 - 5x^2 + 8x - 7$, find $f(6)$.

synthetic substitution (synth. division)
direct substitution (order of operations)
Follow directions

$$\begin{array}{r|rrrrr} 6 & 2 & 0 & -5 & 8 & -7 \\ & \downarrow & 12 & 72 & 402 & 2460 \\ \hline & 2 & 12 & 67 & 410 & 2453 \end{array}$$

$f(6) = 2453$

$x-3$

Use synthetic substitution to find $f(3)$ and $f(-4)$ for each function.

4. $f(x) = x^3 - 2x^2 - x + 1$ **5.** $f(x) = 5x^4 - 6x^2 + 2$

$$\begin{array}{c|cccc} 3 & 1 & -2 & -1 & 1 \\ & \downarrow & 3 & 3 & 6 \\ \hline & 1 & 1 & 2 & 7 \end{array}$$

$$\begin{array}{c|cccc} -4 & 1 & -2 & -1 & 1 \\ & \downarrow & -4 & 24 & -92 \\ \hline & 1 & -6 & 23 & -91 \end{array}$$

$f(3) = 7$

$f(-4) = -91$

How do I know whether something is a factor?
Ex: is 6 a factor of 522?

$$\begin{array}{r} 87 \\ 6\overline{)522} \\ -48 \\ \hline 42 \\ 42 \\ \hline 0 \end{array}$$

yes

$8(87)=522$

$f(4) = 192$

FACTORS OF POLYNOMIALS Divide $f(x) = x^4 + x^3 - 17x^2 - 20x + 32$ by $x - 4$.

$$\begin{array}{r|rrrrr} 4 & 1 & 1 & -17 & -20 & 32 \\ & \downarrow & 4 & 20 & 60 & 160 \\ \hline & 1 & 5 & 3 & 40 & 192 \end{array}$$

$x^3 + 5x^2 + 3x + 40 + \frac{192}{x-4}$

depressed polynomial
factor theorem (R=0)

Key Concept — *Factor Theorem*

The binomial $x - a$ is a factor of the polynomial $f(x)$ if and only if $f(a) = 0$.

Example 2 Use the Factor Theorem

Show that $x + 3$ is a factor of $x^3 + 6x^2 - x - 30$. Then find the remaining factors of the polynomial.

How many more should there be?

$$
\begin{array}{r|rrrr}
-3 & 1 & 6 & -1 & -30 \\
 & \downarrow & -3 & -9 & 30 \\
\hline
 & 1 & 3 & -10 & 0
\end{array}
$$

-10

5 -2

3

$x^2 + 3x - 10$

$(x+3)(x-2)$

$(x+3)\boxed{(x+5)(x-2)}$

$(x+3)\boxed{(x+3)(x-2)}$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials.

6. $x^3 - x^2 - 5x - 3$; $x + 1$

7. $x^3 - 3x + 2$; $x - 1$

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & \downarrow & -1 & 2 & 3 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(x+1)\boxed{(x-3)(x+1)}$

-3

-3 1

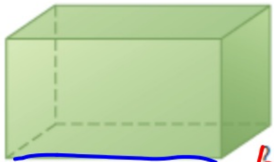
-2

$x^2 - 2x - 3$

$V = B \cdot h$

Example 3 Find All Factors of a Polynomial

GEOMETRY The volume of the rectangular prism is given by $V(x) = x^3 + 3x^2 - 36x + 32$. Find the missing measures.



$x - 4$

$(x-1)$

$(x+8)$

$$\frac{(x-4)(\ ?\ )(\ ?\ )}{x-4} = \frac{x^3 + 3x^2 - 36x + 32}{x-4}$$

$$\begin{array}{r|rrrr} 4 & 1 & 3 & -36 & 32 \\ & \downarrow & 4 & 28 & -32 \\ \hline & 1 & 7 & -8 & 0 \end{array}$$

$x^2 + 7x - 8$

-8, 8, -1, 7

$(x+8)(x-1)$

Fme

4. $\frac{3h^{\frac{2}{5}}}{3} - \frac{9h^{\frac{1}{5}}}{3} + \frac{6}{3} = \frac{0}{3}$

$\left(h^{\frac{2}{5}}\right) - 3\left(h^{\frac{1}{5}}\right) + 2 = 0$

9-350

$u = h^{\frac{1}{5}}$

$u^2 - 3u + 2 = 0$

$(u-1)(u-2) = 0$

$u = 1$

$\left(h^{\frac{1}{5}}\right)^5 = (1)^5$

$h = 1$

$u = 2$

$\left(h^{\frac{1}{5}}\right)^5 = (2)^5$

$h = 32$