Algebra 2 5.5

* Ch. 4 (4.3)

Factor polynomials
Solve polynomial equations by factoring*

factor (expression)
solve (equation)
zero product property
GCF*
perfect square trinomial*
difference of 2 squares*
sum of 2 cubes
difference of 2 cubes
prime polynomial
quadratic form new !!

$$\frac{x^2}{x} + \frac{6xy}{x} - \frac{3yx^3}{x}$$

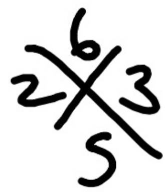
$$x(x + 6y + 3yx^2)$$

$$x^2 + 5x + 6 = 0$$

$$(x+2)(x+3) = 0$$

$x+2=0$ $x+3=0$

$x = -2$ $x = -3$



What is the difference between "factor" and "solve"?

Always look for GCF first...

Example 1 Sum and Difference of Cubes

Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $\frac{16x^4}{2x} + \frac{54xy^3}{2x}$

$2x\,(8x^3 + 27y^3)$

$2x$ $3y$

$2x(2x + 3y)(4x^2 - 6xy + 9y^2)$

$F^3 + S^3 = \underbrace{(F + S)(F^2 - FS + S^2)}$

a   b

$F^3 - S^3 = (F - S)(F^2 + FS + S^2)$

New?

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KeyConcept Sum and Difference of Cubes

Factoring Technique	General Case
Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Wedding song

4.3 Solve by factoring

Example 2 **Factor the GCF**

Solve $16x^2 + 8x = 0.$

$\frac{16x^2}{8x} \quad \frac{8x}{8x}$

$8x(2x+1)=0$

$8x=0 \qquad 2x+1=0$

$\qquad\qquad 2x=-1$

$x=0$

$x=-\frac{1}{2}$

b. $x^2 = 64$

$x^2-64=0$

$(x-8)(x+8)=0$

$x-8=0 \qquad x+8=0$

$x=8$

$x=-8$

Guided Practice

Look for GCF first
Factor vs solve?

1A. $\frac{5y^4}{5y} - \frac{320yz^3}{5y}$ y $4z$

$5y(y^3 - 64z^3)$

$5y(y - 4z)(y^2 + 4yz + 16z^2)$

b. $8y^3 + 5x^3$ prime

$\downarrow$

$2y$

1B. $\frac{-54w^4}{-2w} - \frac{250wz^3}{-2w}$

$-2w(27w^3 + 125z^3)$

$\downarrow \quad \downarrow$

$3w \quad 5z$

$-2w(3w+5z)(9w^2 - 15wz + 25z^2)$

ConceptSummary Factoring Techniques

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Number of Terms	Factoring Technique	General Case
any number	Greatest Common Factor (GCF)	$4a^3b^2 - 8ab = 4ab(a^2b - 2)$
two	Difference of Two Squares	$a^2 - b^2 = (a + b)(a - b)$
	Sum of Two Cubes	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
	Difference of Two Cubes	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
three	Perfect Square Trinomials	$a^2 + 2ab + b^2 = (a + b)^2$ $a^2 - 2ab + b^2 = (a - b)^2$
	General Trinomials factor by grouping	$acx^2 + (ad + bc)x + bd$ $= (ax + b)(cx + d)$
four or more	Grouping	$ax + bx + ay + by$ $= x(a + b) + y(a + b)$ $= (a + b)(x + y)$

Example 2 Factoring by Grouping

Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $[8ax + 4bx + 4cx] + [6ay + 3by + 3cy]$

1. GCF
2. special patterns
3. factor by grouping

$4x(2a+b+c) + 3y(2a+b+c)$

$(2a+b+c)(4x+3y)$

b. $20fy - 16fz + 15gy + 8hz - 10hy - 12gz$

$$(\underbrace{20fy}_{5y} + \underbrace{15gy}_{5y} - \underbrace{10hy}_{5y}) + (\underbrace{-16fz}_{-4z} + \underbrace{8hz}_{-4z} - \underbrace{12gz}_{-4z})$$

$$5y(4f + 3g - 2h) - 4z(4f - 2h + 3g)$$

$$(4f + 3g - 2h)(5y - 4z)$$

GuidedPractice

2A $\left(\frac{30ax}{6x} - \frac{24bx}{6x} + \frac{6cx}{6x}\right) - \frac{5ay^2}{-y^2} + \frac{4by^2}{-y^2} - \frac{cy^2}{-y^2}$

$6x(5a - 4b + c) - y^2(5a - 4b + c)$

$(5a - 4b + c)(6x - y^2)$

Factor completely...

Example 3 Combine Cubes and Squares

Factor each polynomial. If the polynomial cannot be factored, write *prime*.

a. $x^6 - y^6$

$x^2 \quad y^2$

$(x^2 - y^2)(x^4 + x^2y^2 + y^4)$

$(x+y)(x-y)(x^4 + x^2y^2 + y^4)$

GCF
x-factor
difference of squares
difference of cubes
factor by grouping

When to stop?

b $\left(\frac{a^3x^2}{a^3} - \frac{6a^3x}{a^3} + \frac{9a^3}{a^3}\right) + \frac{{}^-b^3x^2}{-b^3} + \frac{6b^3x}{-b^3} - \frac{9b^3}{-b^3}$

$a^3(x^2 - 6x + 9) + {}^-b^3(x^2 - 6x + 9)$

$(x^2 - 6x + 9)(a^3 - b^3)$

$\begin{matrix} 9 \\ \times \\ -6 \end{matrix}$ $(x-3)(x-3)(a-b)(a^2 + \underline{ab + b^2})$

tough…

KeyConcept Quadratic Form

Words	An expression that is in quadratic form can be written as $au^2 + bu + c$ for any numbers a, b, and c, $a \neq 0$, where u is some expression in x. The expression $au^2 + bu + c$ is called the quadratic form of the original expression.

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StudyTip

Quadratic Form When writing a polynomial in quadratic form, choose the expression equal to *u* by examining the terms with variables. Pay special attention to the exponents in those terms. Not every polynomial can be written in quadratic form.

Example 5 Quadratic Form

Write each expression in quadratic form, if possible.

a. $150n^8 + 40n^4 - 15$

2.3.5.5.n.n.n.n.n.n.n.n
2.2.2.5.n.n.n.n
Let u=

b. $y^8 + 12y^3 + 8$

2.3.3.x.x.x.x

Example 6 Solve Equations in Quadratic Form

Solve $18x^4 - 21x^2 + 3 = 0$.

$18x^4 - 21x^2 + 3 = 0$		Original equation
$2(3x^2)^2 - 7(3x^2) + 3 = 0$		$2(3x^2)^2 = 18x^4$
$2u^2 - 7u + 3 = 0$		Let $u = 3x^2$.
$(2u - 1)(u - 3) = 0$		Factor.
$u = \frac{1}{2}$ or	$u = 3$	Zero Product Property
$3x^2 = \frac{1}{2}$	$3x^2 = 3$	Replace u with $3x^2$.
$x^2 = \frac{1}{6}$	$x^2 = 1$	Divide by 3.
$x = \pm\frac{\sqrt{6}}{6}$	$x = \pm 1$	Take the square root.

The solutions of the equation are $1, -1, \frac{\sqrt{6}}{6}$, and $-\frac{\sqrt{6}}{6}$.

‣ **Guided**Practice

6A. $4x^4 - 8x^2 + 3 = 0$

2.2.x.x.x.x
2.2.2.x.x

2.2.2.x.x.x.x
2.5.x.x

6B. $8x^4 + 10x^2 - 12 = 0$