Algebra 2 7.8

Use logarithms to solve problems involving exponential growth and decay

Use logarithms to solve problems of logistic growth

Calculate half-life

$$y = a e^{kt}$$

half-life

carbon dating

logistic growth

KeyConcept Exponential Growth and Decay

Exponential Growth	Exponential Decay
Exponential growth can be modeled by the function $f(x) = ae^{kt}$, where a is the initial value, t is time in years, and k is a constant representing the rate of continuous growth.	Exponential decay can be modeled by the function $f(x) = ae^{-kt}$, where a is the initial value, t is time in years, and k is a constant representing the rate of continuous decay.

half-life = length of TIME
If you start with 100%...

Real-World Example 1 Exponential Decay

SCIENCE The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to disintegrate. The half-life of Carbon-14 is 5730 years. Determine the value of k and the equation of decay for Carbon-14.

$k \cdot 5730$

$\frac{50}{100} = \frac{100e}{100}$

$\ln 0.5 = \ln e^{5730k}$

$y = ae^{-0.00021T}$

$-0.6931 = 5730k\,(1)$

$-0.000121 = k$

GuidedPractice

1. The half-life of Plutonium-239 is 24,000 years. Determine the value of k.

So if it was 100% then...

Real-World Example 2 Carbon Dating

SCIENCE A paleontologist examining the bones of a prehistoric animal estimates that they contain 2% as much Carbon-14 as they would have contained when the animal was alive.

a. How long ago did the animal live?

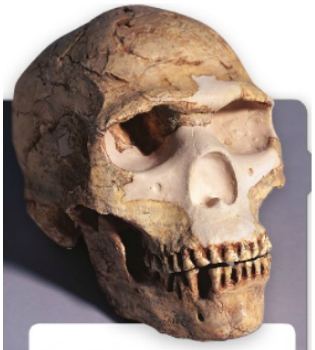
(Use k=-.00012 from ex. 1)

$$2 = 100e^{-0.00012T}$$

$$\ln 0.02 = \ln e^{-0.00012T}$$

$$-3.9120 = -0.00012T \quad (1)$$

$$T \approx 32{,}600$$



Real-World Link

The oldest modern human fossil, found in Ethiopia, is approximately 160,000 years old.

Source: National Public Radio

b. If prior research points to the animal being around 20,000 years old, how much Carbon-14 should be in the animal?

StudyTip

Carbon Dating When given a percent or fraction of decay, use an original amount of 1 for *a*. (100%)

Prob. 9 (from yesterday)

100 mg U-238 $\quad$ HL = 4,47 bill.

$\downarrow$ $\quad$ k(4.47)

10 mg.

$\frac{50}{100} = \frac{100e}{100}$

$\ln 0.5 = \ln e^{4.47k}$

$-0.6931 = 4.47k$

$-0.1551 = k$

$y = ae^{-0.1551T}$

$\frac{10}{100} = \frac{100e^{-0.1551T}}{100}$

$\ln 0.1 = \ln e^{-0.1551T}$

$-2.3026 = -0.1551T$

$14.85 = T$

$y = ae^{kT}$

$\frac{3.7}{3.4} = \frac{3.4e^{k \cdot 6}}{3.4}$

$\ln 1.0882 = \ln e^{6k}$

$0.08456 = 6k$

$k = 0.0141$

$y = ae^{0.0141T}$

$y = 3.4e^{0.0141 \cdot 20}$

$\doteq 4.5$ million

$6 = 3.4e^{0.0141T}$

$\ln 1.7647 = \ln e^{0.0141T}$

$0.5680 = 0.0141T$

≈ 40 yrs

2040

Use k from ex. 1

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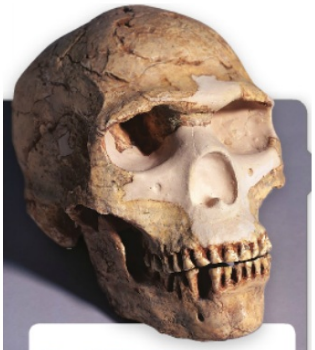

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GuidedPractice

2. Use the information in Example 2 to answer the following questions.

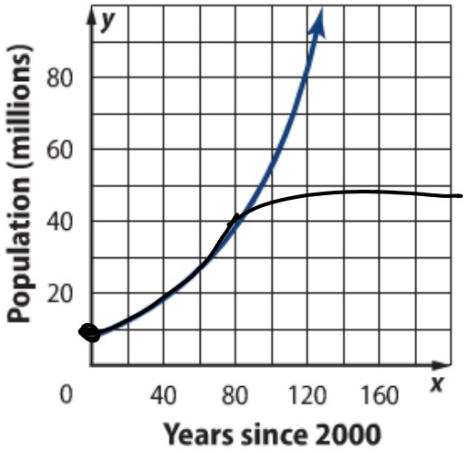
A. A specimen that originally contained 42 milligrams of Carbon-14 now contains 8 milligrams. How old is the fossil?

B. A wooly mammoth specimen was thought to be about 12,000 years old. How much Carbon-14 should be in the animal?

2 Logistic Growth Refer to the equation representing Georgia's population in Example 3. According to the graph at the right, Georgia's population will be about one billion by the year 2130. Does this seem logical?

Populations cannot grow infinitely large. There are limitations, such as food supplies, war, living space, diseases, available resources, and so on.

Exponential growth is unrestricted, meaning it will increase without bound. A **logistic growth model**, however, represents growth that has a limiting factor. Logistic models are the most accurate models for representing population growth.



limiting factor...

KeyConcept Logistic Growth Function

Let a, b, and c be positive constants where $b < 1$. The logistic growth function is represented by $f(t) = \frac{c}{1 + ae^{-bt}}$, where t represents time.

Will give you this equation if needed

Graphing calculator (be careful with parentheses)

Real-World Example 4 Logistic Growth

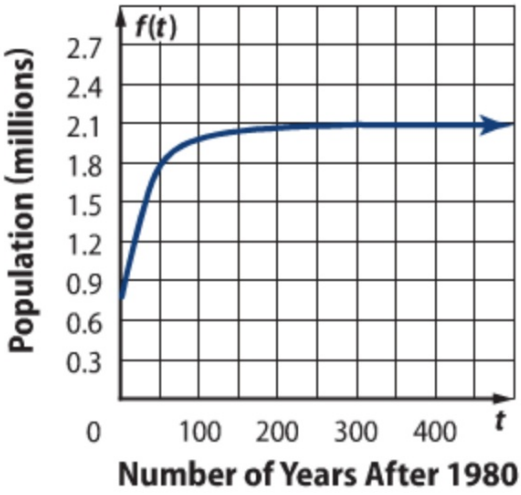
The population of Phoenix, Arizona, in millions can be modeled by the logistic function $f(t) = \frac{2.0666}{1 + 1.66e^{-0.048t}}$, where t is the number of years after 1980.

a. Graph the function for $0 \leq x \leq 500$.

b. What is the horizontal asymptote?

$y \approx 2.0666$ million

better graph next page


f(t)
2.7
2.4
2.1
1.8
1.5
1.2
0.9
0.6
0.3
Population (millions)
0
100
200
300
400
t
Number of Years After 1980

c. Will the population of Phoenix increase indefinitely? If not, what will be their maximum population?

d. According to the function, when will the population of Phoenix reach 1.8 million people?

50.5 yrs

approx 2030

GuidedPractice

4. The population of a certain species of fish in a lake after t years can be modeled by the function $P(t) = \dfrac{1880}{1 + 1.42e^{-0.037t}}$, where $t \geq 0$.

A. Graph the function for $0 \leq t \leq 500$.

B. What is the horizontal asymptote?

C. What is the maximum population of the fish in the lake?

D. When will the population reach 1875?

b. If prior research points to the animal being around 20,000 years old, how much Carbon-14 should be in the animal?

StudyTip

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