Algebra 2 8.4
Graph rational functions with vertical and horizontal asymptotes
Graph rational functions with oblique asymptotes
Graph rational functions with point discontinuity

repeat → point discont. 0 = open

rational function
zero (of a function)
vertical asymptote
horizontal asymptote
oblique (slant) asymptote
point discontinuity

VA x =

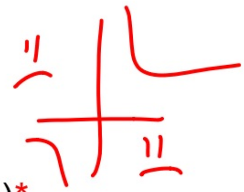
HA | SA

$\frac{\text{num}}{\text{denom}}$ = y = ratio

$\overline{\text{denom}\uparrow}$ y = 0

$\overline{\text{denom}\downarrow}$ SA if one

Consider:



VA (denom)
HA (look at degree)*
SA (degree)*
Point discontinuity (cancelled factors...denom)

*One at most

x-intercepts (if any)
y-intercepts (if any)

A few ordered pairs
Must be a function (vert. line test)

(ratio)

Whiteboards

$(x-1)(x^2+x+1)$

3B. $f(x) = \dfrac{x^3 - 1}{x^2 - 4}$

$(x-2)(x+2)$

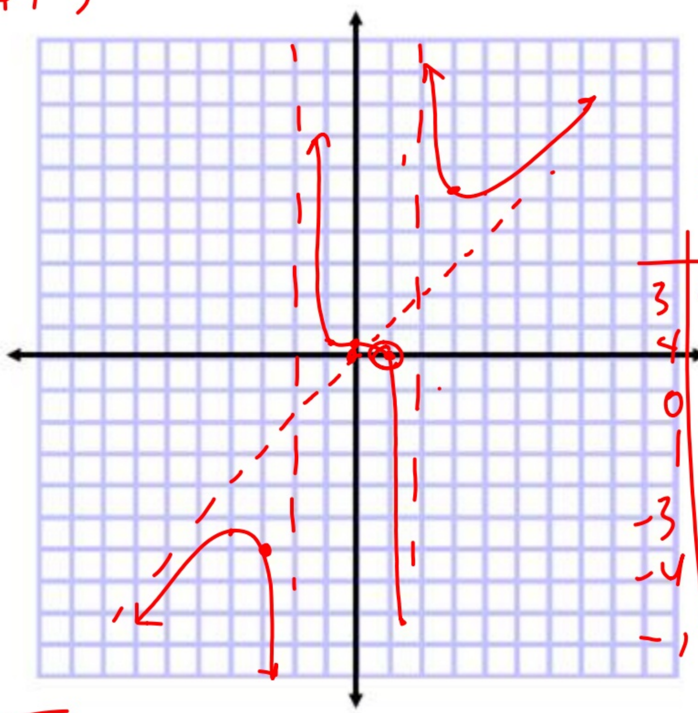
$x-2=0$ $\quad$ $x+2=0$

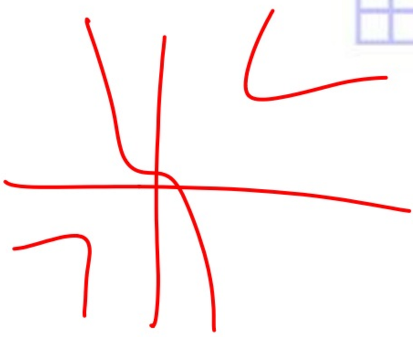
$x=2$ $\quad$ $x=-2$

SA $y=x$

VA

$$\begin{array}{r|l} & x \\ x^2-4 & x^3 \qquad -1 \\ & -x^3+4x \\ \hline & 4x-1 \end{array}$$





$\dfrac{x^3-1}{x^2-4}$

3	$\frac{26}{5}$
4	$\frac{63}{12}$
0	$\frac{-1}{-4}$
1	$\frac{0}{-4}$
-3	$\frac{-28}{5}$
-4	$\frac{-65}{12}$
-1	$\frac{-2}{-3}$

$x = -1$

$x + 1 = 0$

$-1 \quad -1$

Repeated factors = point discontinuity

KeyConcept Point Discontinuity

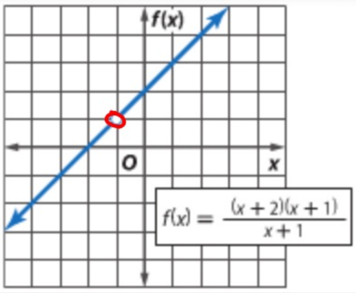
Words If $f(x) = \frac{a(x)}{b(x)}$, $b(x) \neq 0$, and $x - c$ is a factor of both $a(x)$ and $b(x)$, then there is a point discontinuity at $x = c$.

Example

$$f(x) = \frac{(x+2)(x+1)}{x+1}$$

$$= x + 2;\ x \neq -1$$

$y = x + 2$



If something "cancels out" of original equation

Example 4 Graph with Point Discontinuity

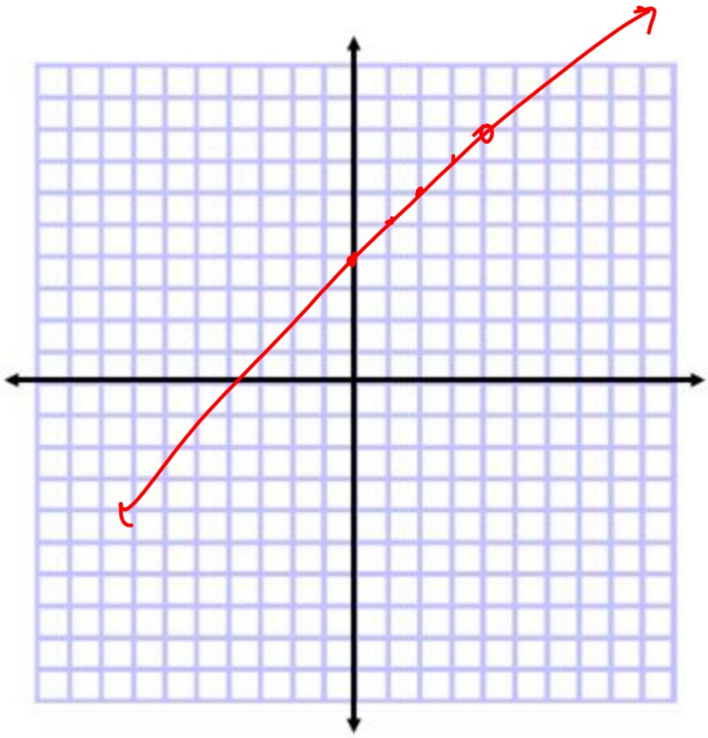
$\cancel{(x-4)}(x+4)$

Graph $f(x) = \dfrac{x^2 - 16}{\cancel{x - 4}}$.

$x - 4 = 0$
$x = 4$ P.D.

Is it an asymptote or a point discontinuity?

$f(x) = x + 4$



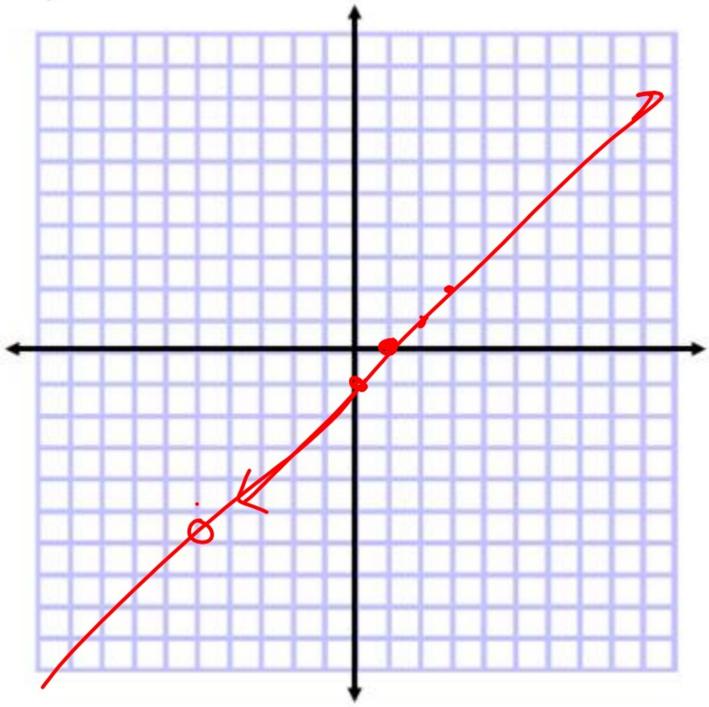
$(\cancel{x+5})(x-1)$

Graph each function.

4A. $f(x) = \dfrac{x^2 + 4x - 5}{\cancel{x + 5}}$

P.D $x = -5$

$y = x - 1$



$$(\cancel{x+3})(\cancel{x-3})(x+2)$$

4B. $f(x) = \dfrac{x^3 + 2x^2 - 9x - 18}{x^2 - 9}$

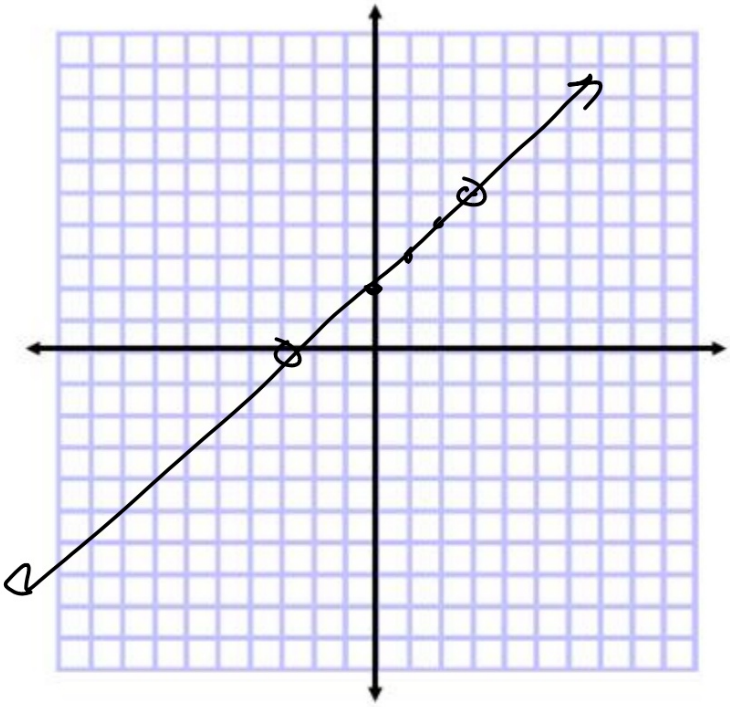
$$(\cancel{x+3})(\cancel{x-3})$$

P.D. $x=-3$ $x=3$

$$(x^3+2x^2) - \underset{=9}{9}x - \underset{=9}{18}$$

$$x^2(x+2) - 9(x+2)$$

$$(x^2-9)(x+2)$$



$T = 6 \text{ hrs}$ $\quad \frac{1}{6}$

$F = x \text{ hrs}$ $\quad \frac{1}{\cancel{4}}$

$T + F = \text{less}$ $\quad \frac{1}{6+x}$

	$\frac{6+x}{6x}$	
1	$\frac{7}{6}$	1.2
2	$\frac{8}{12}$	$\frac{2}{3}$
3		

$$f(x) = \frac{6+x}{6x}$$

$\frac{10}{24}$ $\quad \frac{5}{12}$

VA $\quad 6x = 0$
$x = 0$

HA $\quad \frac{x^1}{x^1}$, $y = 1$