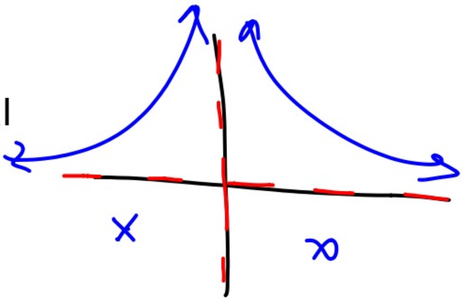
$\boxed{y = \frac{1}{x}}$



Algebra 2 8.4

Graph rational functions with vertical and horizontal asymptotes

Graph rational functions with oblique asymptotes

Graph rational functions with point discontinuity

rational function
zero (of a function)
vertical asymptote
horizontal asymptote
oblique (slant) asymptote
point discontinuity

1 Vertical and Horizontal Asymptotes

A **rational function** has an equation of the form $f(x) = \frac{a(x)}{b(x)}$, where $a(x)$ and $b(x)$ are polynomial functions and $b(x) \neq 0$.

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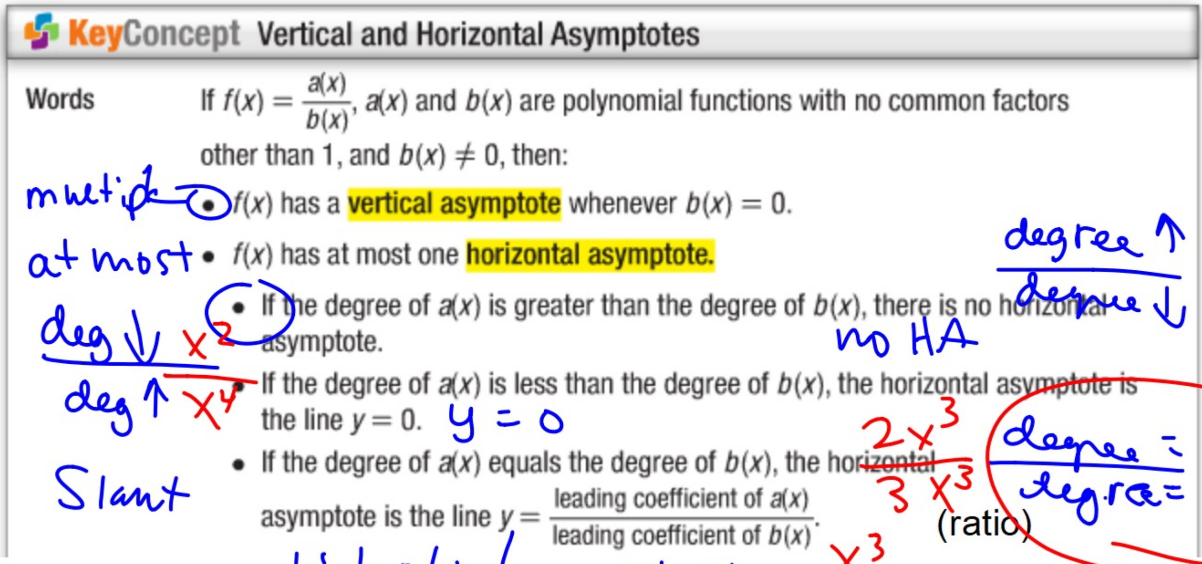


KeyConcept Vertical and Horizontal Asymptotes

Words If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1, and $b(x) \neq 0$, then:

multiple → - $f(x)$ has a **vertical asymptote** whenever $b(x) = 0$.

at most - $f(x)$ has at most one **horizontal asymptote.**

$\frac{\text{degree} \uparrow}{\text{degree} \downarrow}$ $\frac{x^3}{x}$

$\frac{\text{deg} \downarrow}{\text{deg} \uparrow}$ $\frac{x^2}{x^4}$

  - If the degree of $a(x)$ is greater than the degree of $b(x)$, there is no horizontal asymptote. no HA
  - If the degree of $a(x)$ is less than the degree of $b(x)$, the horizontal asymptote is the line $y = 0$. $y = 0$
  - If the degree of $a(x)$ equals the degree of $b(x)$, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } a(x)}{\text{leading coefficient of } b(x)}$.

$\frac{2x^3}{3x^3}$ (ratio) $\frac{\text{degree} =}{\text{degree} =}$ } ratio

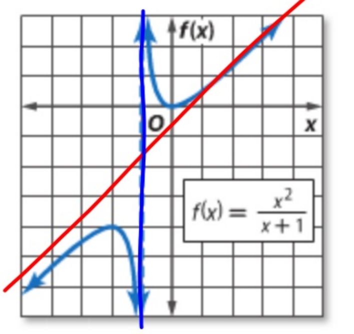
Slant

higher by 1 slant $\frac{x^3}{x^2}$

Examples

$\frac{x^2}{x+1}$

No horizontal asymptote

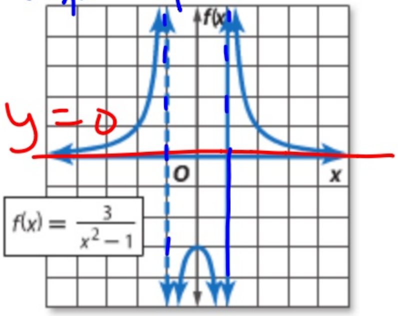


Vertical asymptote:
$x = -1$

$\frac{3}{x^2-1}$

$(x+1)(x-1)$

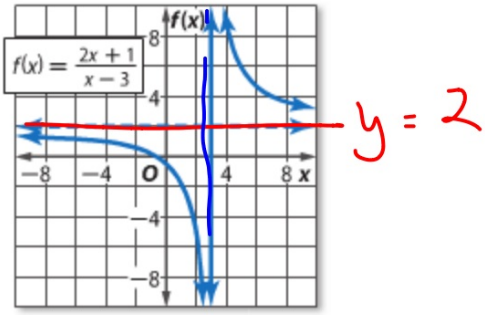
One horizontal asymptote



Vertical asymptotes:
$x = -1$, $x = 1$
Horizontal asymptote:
$f(x) = 0$

$\frac{2x+1}{x-3}$ $\frac{2x}{x}$ $\frac{2}{1}$

$x-3=0$

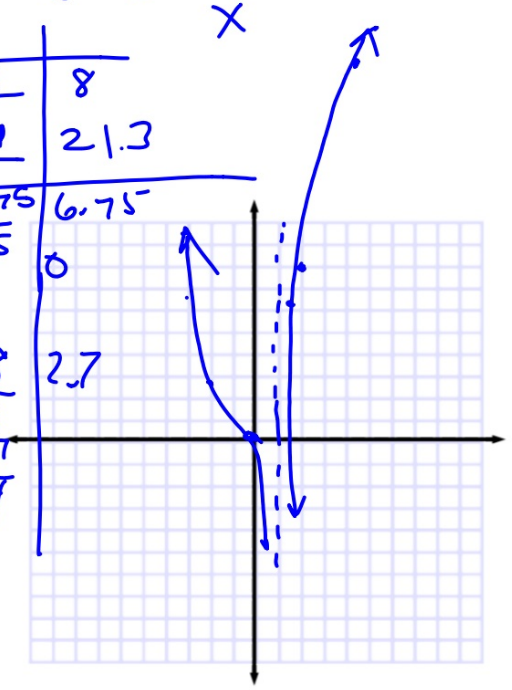


Vertical asymptote:
$x = 3$
Horizontal asymptote:
$f(x) = 2$

Example 1 Graph with no Horizontal Asymptote

Graph $f(x) = \frac{x^3}{(x-1)}$ $\frac{x^3}{x}$

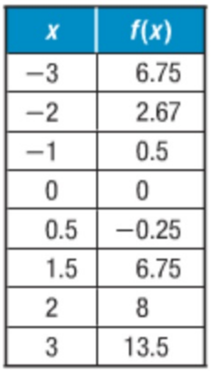
2	$\frac{8}{1}$	8
4	$\frac{64}{3}$	21.3
1.5	$\frac{3.375}{.5}$	6.75
0	$\frac{0}{-1}$	0
-2	$\frac{-8}{-3}$	2.7
-3	$\frac{-27}{-4}$	



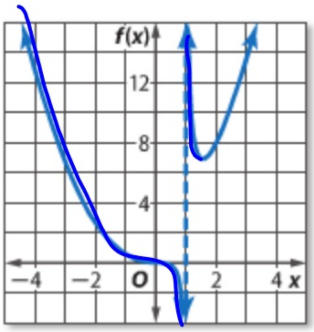
VA $x-1=0$ $x=1$

1. find the vertical asymptotes (denom=0)
2. find the horizontal asymptotes (if any)
   ...you can tell by degree of num/denom
3. Plot a few ordered pairs (each region)
   (find zeros (where numer=0))

next



x	f(x)
−3	6.75
−2	2.67
−1	0.5
0	0
0.5	−0.25
1.5	6.75
2	8
3	13.5



2 Oblique Asymptotes and Point Discontinuity

An **oblique asymptote**, sometimes called a *slant asymptote*, is an asymptote that is neither horizontal nor vertical.

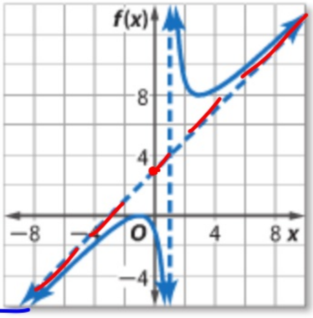
KeyConcept Oblique Asymptotes

Words If $f(x) = \frac{a(x)}{b(x)}$, $a(x)$ and $b(x)$ are polynomial functions with no common factors other than 1 and $b(x) \neq 0$, then $f(x)$ has an oblique asymptote if the degree of $a(x)$ minus the degree of $b(x)$ equals 1. The equation of the asymptote is $f(x) = \frac{a(x)}{b(x)}$ with no remainder.

Example $f(x) = \frac{x^4 + 3x^3}{x^3 - 1}$

Vertical asymptote: $x = 1$

Oblique asymptote: $f(x) = x + 3$



$y = x + 3$

$\boxed{x+3}$

$x^3 - 1 \overline{)\, x^4 + 3x^3}$

$-x^4 + x$

$3x^3 + x$

$-3x^3 + 3$

~~$x + 3$~~

Horizontal or slant (not both)
Long division ans

~~point discontin.?~~

~~-6 -3 2 -1~~

8, 4   9-25 odd

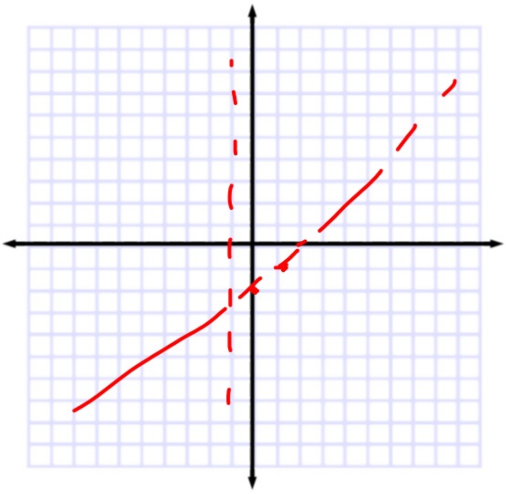
Guided Practice

Graph each function.

1A. $f(x) = \frac{x^2 - x - 6}{x + 1}$ *

$y = x - 2$

$$
\begin{array}{r|l}
 & \boxed{x - 2} \\
x + 1 & x^2 - x - 6 \\
 & -\,x^2 \mp x \\
\hline
 & -2x - 6 \\
 & +2x + 2 \\
\hline
 & -4
\end{array}
$$



1. find the vertical asymptotes
2. find the horizontal asymptotes (if any)
3. find the slant asymptotes (if any)
4. Consider zeros: Plot a few ordered pairs

Example 3 Determine Oblique Asymptotes

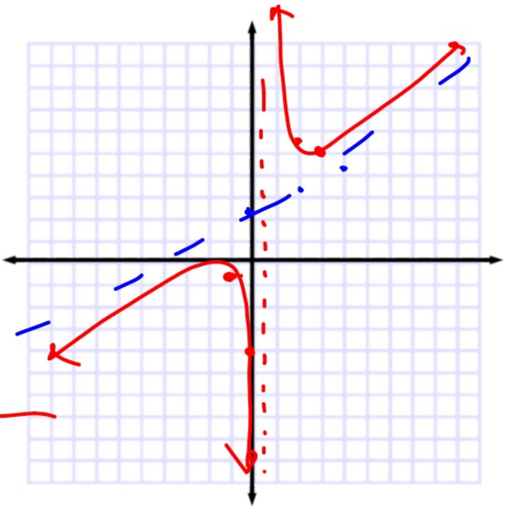
Graph $f(x) = \frac{x^2 + 4x + 4}{2x - 1}$.

$$\boxed{y = \frac{1}{2}x + 2.25}$$

$$\frac{1}{2}x + 2.25$$

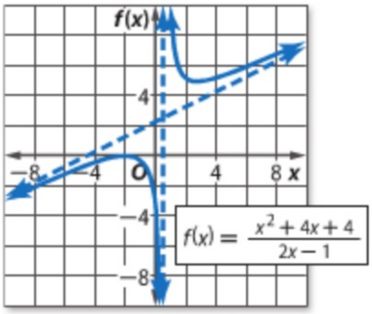
$2x - 1$

3	$\frac{9+12+4}{6-1}$	5
2	$\frac{4+8+4}{4-1}$	5.3
0	$\frac{0+0+4}{-1}$	-4
-1	$\frac{1+{}^-4+4}{{}^-2-1}$	$\frac{1}{-3}$



1. find the vertical asymptotes
2. find the horizontal asymptotes (if any)
3. find the slant asymptotes (if any)
4. Consider zeros: Plot a few ordered pairs (each region)

Draw the asymptotes, and then use a table of values to graph the function.



GuidedPractice

Graph each function.

$$\begin{array}{r} x+2 \\ x-2\,\overline{)\,x^2} \\ -x^2+2x \end{array}$$

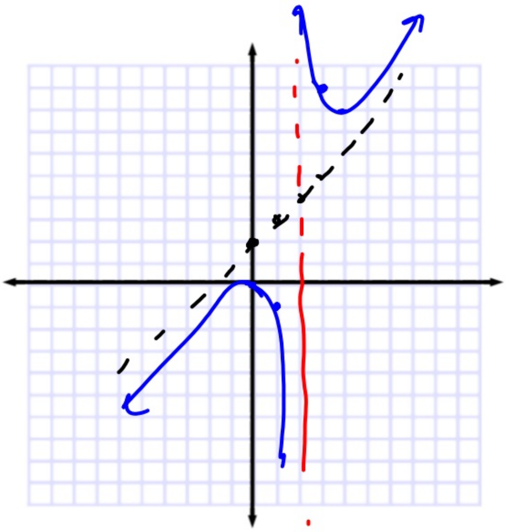
3A. $f(x) = \frac{x^2}{x-2}$

$x-2=0$
$x=2$

$y = x+2$

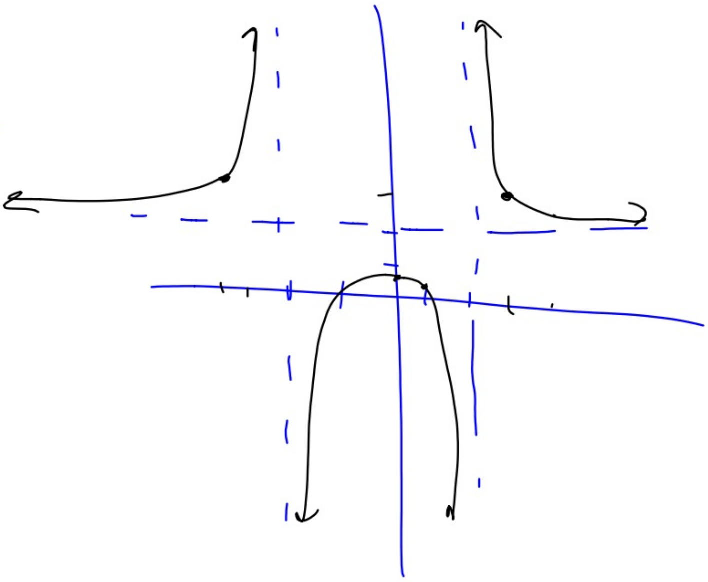
3	$\frac{9}{1}$	9
4	$\frac{16}{2}$	8
1	$\frac{1}{-1}$	-1
0	$\frac{0}{-2}$	0

1. find the vertical asymptotes
2. find the horizontal asymptotes (if any)
3. find the slant asymptotes (if any)
4. Consider zeros: Plot a few ordered pairs (each region)



$$y = \frac{2x^2 - 3}{x^2 - 4}$$

3	$\frac{2 \cdot 9 - 3}{9 - 4}$	$\frac{15}{5}$	3
4	$\frac{2 \cdot 16 - 3}{16 - 4}$	$\frac{29}{12}$	2.4
1	$\frac{2 - 3}{1 - 4}$	$\frac{-1}{-3}$	$\frac{1}{3}$
0	$\frac{0 - 3}{-4}$		$\frac{3}{4}$
-3	$\frac{18-3}{9-4}$	$\frac{15}{5}$	3
-4			



2
1 2
3 PT discont.

factor/cancel

open @ x =

$$(\cancel{x+1})\boxed{(x+2)}$$

$$y = \frac{x^2+3x+2}{\cancel{x+1}}$$

$x = -1$

$y = x + 2$