

Trig 3.7

Graph rational functions

Determine horizontal, vertical, slant asymptotes

rational function $y=1/x$

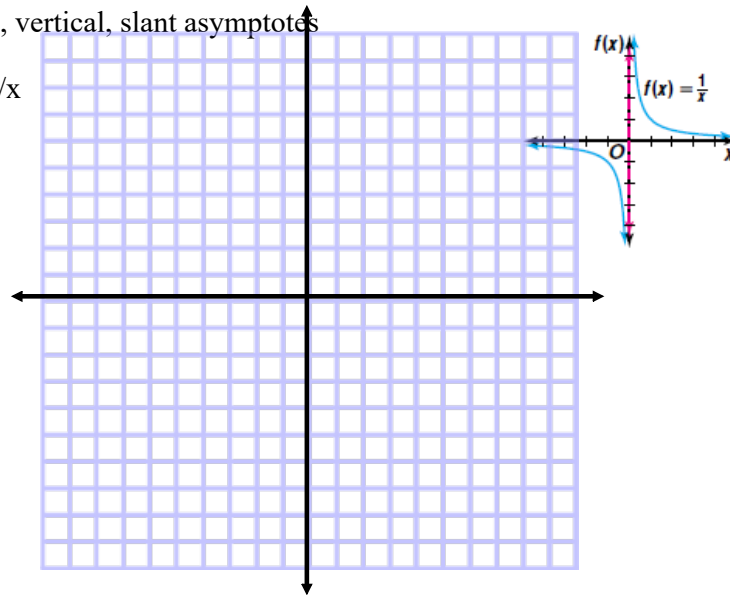
vertical asymptote

horizontal asymptote

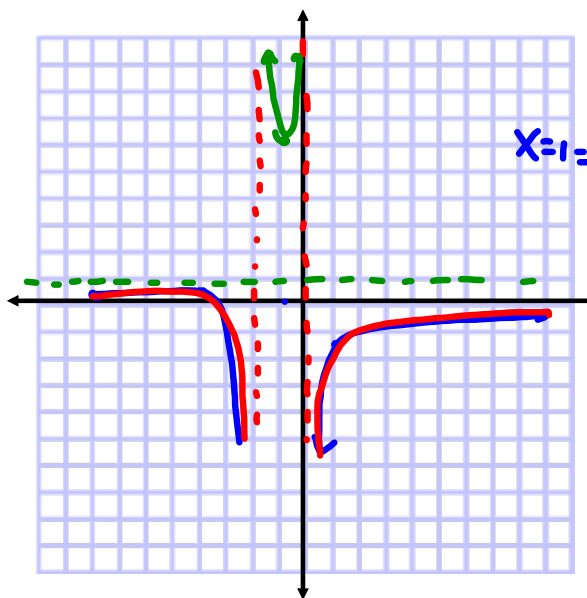
point discontinuity

slant asymptote

common factor



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$$y = \frac{(x-2)(x+3)}{x(x+2)}$$

$$\begin{array}{r} x^2 - x - 6 \\ \hline x^2 + 2x \end{array}$$

$$x=1 = \frac{(-1)(4)}{1(3)} = -\frac{4}{3}$$

$$x=0$$

$$x+2=0$$

$$x=-4 = \frac{-6(-1)}{-4(-2)}$$

$$x=-2$$

$$x=-1 = \frac{(-2)(2)}{(-1)(1)} = \frac{-4}{-1} = 4$$

VA (vars in denom) \rightarrow makes denom = 0

HA $y=0$ if d-denom is greater $\frac{x^3}{x^4} \rightarrow \frac{2x^2}{3x^2}$

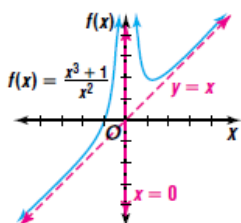
• $y = \text{ratio}$ if d-num=d-denom
 no HA if d-num > d-denom $\frac{x^4}{x^2}$

• SA (if degree of num > degree denom by exactly 1)

old school long \div

$$\frac{x^3}{x^2}$$

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A third type of asymptote is a **slant asymptote**. Slant asymptotes occur when the degree of the numerator of a rational function is *exactly one greater* than that of the denominator. For example, in $f(x) = \frac{x^3 + 1}{x^2}$, the degree of the numerator is 3 and the degree

of the denominator is 2. Therefore, this function has a slant asymptote, as shown in the graph. Note that there is also a vertical asymptote at $x = 0$. *When the degrees are the same or the denominator has the greater degree, the function has a horizontal asymptote.*

$$y = x$$

Find SA using long division (old school)

$$\begin{array}{r} x \overline{) x^3 + 1} \\ \underline{-x^3} \\ 1 \end{array}$$

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VA $x = 0$
~~HA~~
 SA $y = x - 1$

Graph each function.
 35. $y = \frac{(x-2)(x+1)}{x}$

if $x = 2$ $\frac{0 \cdot 3}{2} = \frac{0}{2} = 0$
 if $x = -1$ $\frac{(-3) \cdot 0}{-1} = \frac{0}{-1} = 0$

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36. $y = \frac{(x^2 - 4)(\cancel{x-2})(x+2)}{(x-2)(\cancel{x-2})}$
 $y = x + 2$

Common factor...
~~VA~~ point discon.
~~HA~~
~~SA~~
 Hum... $x = 2$

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39. $y = \frac{x^2 - 6x + 9}{x^2 - x - 6}$

$(x-3)(x-3)$
 $(x-3)(x+2)$

if $x = 0$ $-\frac{3}{2} = -1.5$
 if $x = -4$ $-\frac{-7}{-2} = 3.5$

$y = \frac{x-3}{x+2}$

Consider (some or all):

VA $x = -2$
 HA $y = 1$
 SA

crossing points
 common factors

Hmmm $x = 3$

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40. $y = \frac{x^2 - 1}{x^2 - 2x + 1}$

$(x-1)(x+1)$
 $(x-1)(x-1)$

$x = 2$ $\frac{3}{1} = 3$
 $x = 0$ $\frac{1}{-1} = -1$

$y = \frac{x+1}{x-1}$

Consider (some or all):

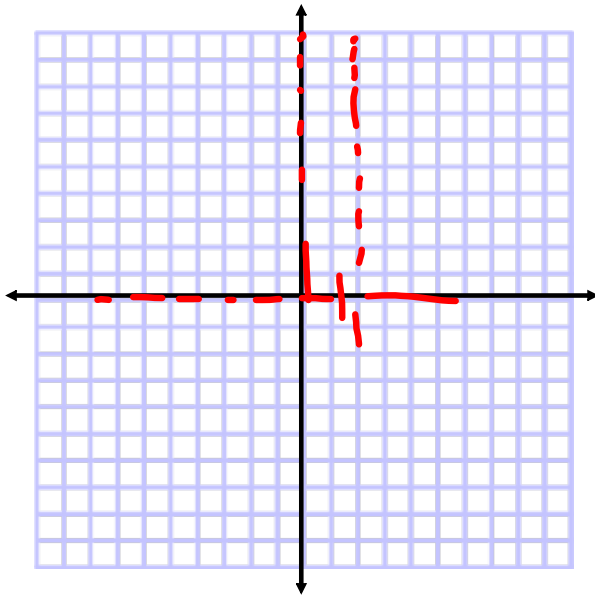
VA $x = 1$
 HA $y = 1$
~~SA~~

~~crossing points~~
 common factors

Hmmm... $x = 1$

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5 Graph $y = \frac{(x+3)(x+1)}{x(x+3)(x-2)}$ $\frac{x^2}{x^3}$



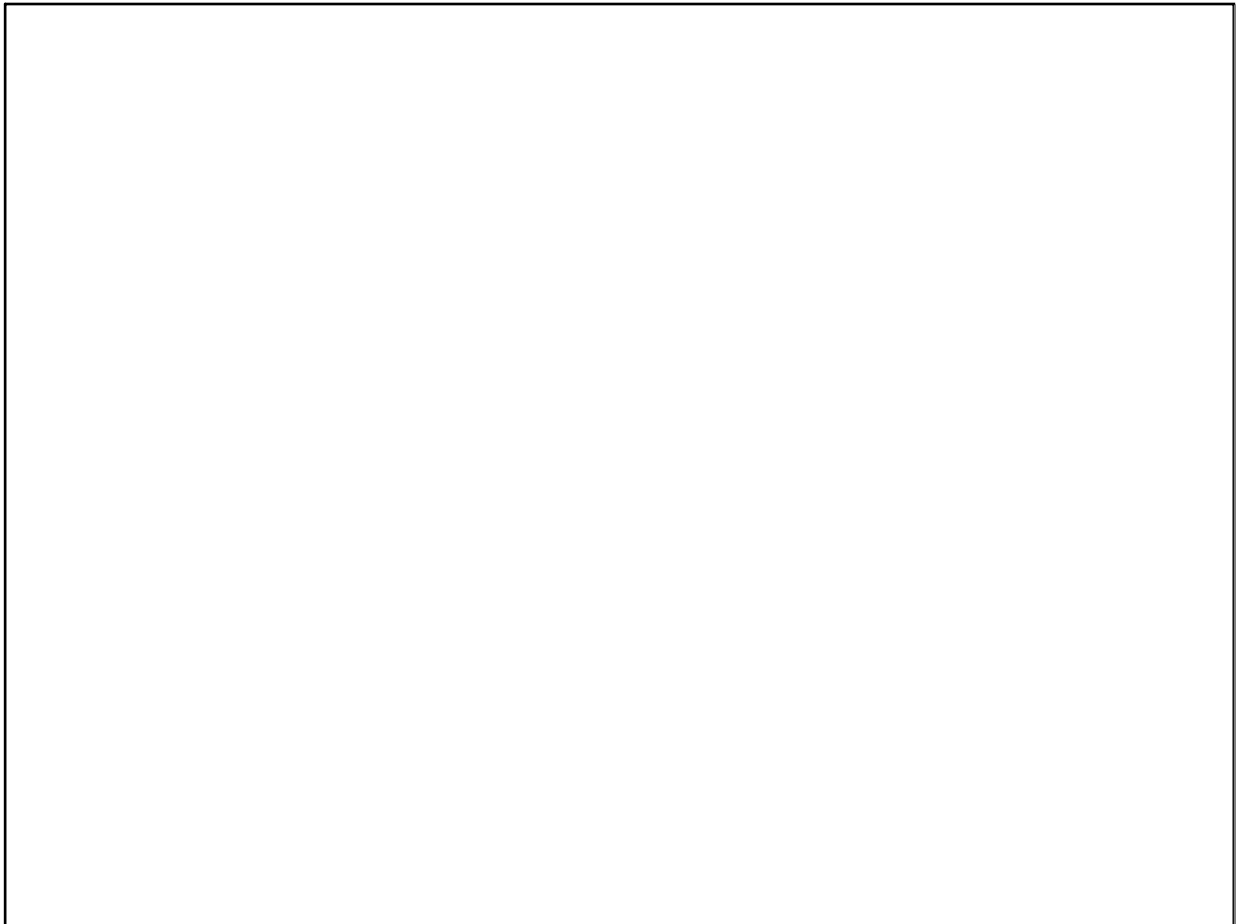
Consider (some or all):

VA $x=0$ $x=2$
 HA $y=0$
 crossing points
 common factors

Hmm $x = -3$

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 PA-3 | (1-4)
 EP

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