

Precalc 10.7

Find the equations of conic sections that have been rotated or translated

Graph rotations and/or translations of conics

Use the discriminant to identify conic sections

Find the angle of rotation for a given equation

rotation

Quiz 10.5-10.6

Bxy

translation

discriminant

degenerate case (disregard)

Graphing calculator

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FMC ①

$$1. 2(x-3)^2 + 3(x-3)(y-1) - 4(y-1)^2 - 5(x-3) = 0$$

$$2(x^2 - 6x + 9) + 3(xy - 3y - x + 3) - 4(y^2 - 2y + 1)$$

$$- 5(x-3) = 0$$

$$\underline{2x^2} - \underline{12x} + \underline{18} + \underline{3xy} - \underline{9y} - \underline{3x} + \underline{9} - \underline{4y^2} + \underline{8y} - \underline{4} - \underline{5x} + \underline{15} = 0$$

$$2x^2 + 3xy - 4y^2 - 20x - y + 38 = 0$$

$$x^2 + 4xy + 5y^2 - 7x - y = 0$$

$$(y')^2 + 4(y')(-x') + 5(-x')^2 - 7(y') - (-x') = 0$$

$$(y')^2 - 4x'y' + 5(x')^2 - 7(y') + x' = 0$$

$$\tan 2\theta = \frac{B}{A-C} = \frac{4}{1-5} = \frac{4}{-4} = -1$$

$$\tan 2\theta = -1$$

$$2\theta = -45^\circ$$


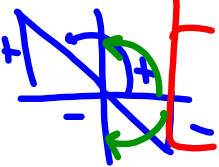
$$\theta = -22.5^\circ$$

$$x \rightarrow x' \cos \theta + y' \sin \theta$$

$$y \rightarrow -x' \sin \theta + y' \cos \theta$$

$$x \rightarrow y'$$

$$y \rightarrow -x'$$

$$x^2 - 5x + y^2 = 3$$

$$0 - 4 \cdot 1 \cdot 1 = -4$$

$$\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right)^2 - 5\left(\frac{1}{2}x' + \frac{\sqrt{3}}{2}y'\right) + \left(-\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'\right)^2 = 3$$

$$x \rightarrow x' \cos \frac{1}{2}60 + y' \sin \frac{1}{2}60$$

$$y \rightarrow -x' \sin \frac{\sqrt{3}}{2}60 + y' \cos \frac{1}{2}60$$

$$x \rightarrow \frac{1}{2}x' + \frac{\sqrt{3}}{2}y'$$

$$y \rightarrow -\frac{\sqrt{3}}{2}x' + \frac{1}{2}y'$$

$$\frac{1}{4}(x')^2 + \frac{\sqrt{3}}{2}x'y' + \frac{3}{4}(y')^2 - \frac{5}{2}x' - \frac{5\sqrt{3}}{2}y' + \frac{3}{4}(x')^2 - \frac{\sqrt{3}}{2}x'y' + \frac{1}{4}(y')^2 = 3$$

$$(x')^2 + (y')^2 - \frac{5}{2}x' - \frac{5\sqrt{3}}{2}y' - 3 = 0$$

$$2(x')^2 + 2(y')^2 - 5x' - 5\sqrt{3}y' - 6 = 0$$

To find the equation of a conic section with respect to a rotation of θ , replace

x with $x' \cos \theta + y' \sin \theta$
and y with $-x' \sin \theta + y' \cos \theta$.

If xy term: all bets are off...

Discriminant

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Identifying
Conics By
Using the
Discriminant

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$,

- if $B^2 - 4AC < 0$, the graph is a circle ($A = C, B = 0$) or an ellipse ($A \neq C$ or $B \neq 0$);
- if $B^2 - 4AC > 0$, the graph is a hyperbola;
- if $B^2 - 4AC = 0$, the graph is a parabola.

$$\frac{b^2 - 4ac}{2a}$$

$$B^2 - 4AC$$

neg (circle/ellipse)
pos (hyp)
0 (para)

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Angle of
Rotation
About the
Origin

For the general equation $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, the angle of rotation θ about the origin can be found by

$$\left(\begin{array}{l} \theta = \frac{\pi}{4}, \text{ if } A = C, \text{ or} \\ \tan 2\theta = \frac{B}{A - C}, \text{ if } A \neq C. \end{array} \right)$$

$$\tan 2\theta = \frac{B}{A - C}$$

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Questions?

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